Intergenerational redistribution in representative democracies

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Intergenerational Redistribution in Representative Democracies

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Proefschrift

ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof.dr. L.F.W. de Klerk, in het openbaar te verdedigen ten overstaan van een door het college van dekanen aangewezen commissie in de aula van de Universiteit op maandag 6 mei 1996 om 14.15 uur

door

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Writing a PhD thesis can to some extent be compared to riding a stage race in cycling. The prologue should give a first impression of the condition of the rider. The various stages are covered in the following chapters. The final stage, when the overall score is usually settled, gives the possibility to look back and discuss the race.

However, to have the opportunity to start at all, one needs to join a team, which was the Public Finance group at Tilburg University. Therefore, first, I want to thank its team manager, Harrie Verbon for letting me join his team and giving me this opportunity. At a later stage, Casper van Ewijk joined the team management and was a great stimulation when completing the last stages.

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Finally, I would like to thank my friends and family for showing interest in what I was doing without most of them ever having the faintest idea about what I was really doing. Most likely, after reading this preface and attempting to read the rest of this thesis, this state continues. But now at least they know that I passed the finishing line.

Contents

Chapter 1

Prologue

1.1 Introduction

Many government policies redistribute resources between generations: public pensions, public health care, public debt, investment in public education or infrastructure. In the Netherlands for example, the contribution rate for the Pay-As-You-Go public pension system is 14% of total earnings of an individual to a maximum of fl. 6000,– per year. The total contributions amount to about 6% of GDP. This is a pure redistribution from the working population to the elderly. In case of public health care, the costs for an 85-year old are approximately 9 times as high as for an 45-year old. However, the contribution of an 85-year old does certainly not offset this. Investment in public education, which redistributes from current working and retired to the youngest generations, is approximately 5% of GDP. In the 70s and 80s a large increase in debt levels occured (from a debt/GDP ratio of about 40% in 1975 to almost 80% in 1995) implying a transfer of resources from the future to the present. The situation in the Netherlands does not differ much from the situation in other Western countries. Taking all the different redistributive policies at some point in time into account, it can be calculated which generations are the net recipients and which are the net contributors at that point in time. The youngest generations, *i.e.* those to the age of about 20, and the oldest generations, *i.e.* the pensioners, are the net recipients. The younger generations hardly pay taxes but do benefit from expenditures on child benefits and education. The elderly, compared to benefits they receive in the form of pensions or public health care, pay less than an equivalent amount of taxes. The working generations, who are the major tax payers, are the net contributors.

Though real data on redistributive flows between generations are lacking, the numbers above give an impression of the flow of resources between the different generations at a certain point in time. One step further is to calculate the total of transfers received and payed over the lifecycle, *i.e.* the generational accounts. Calculations for the US by Auerbach et al. (1994) indicate that tax rates have to rise considerably in the future if the government wants to obey its intertemporal budget constraint. This indicates a redistribution from the future to the present. Calculations by the OECD for some other countries[1] give similar results. Considering public pensions, public health care, public spending on education and child benefits, it appears that for the Netherlands, generations born between 1935 and 1955 are paying more than they receive, with a minimum for the benefit/cost-ration of about 0.7. The opposite holds for generations born before 1935 and after 1955, with a maximum for the benefit/cost-ration of about 1.3. Due to the ageing of the population, generations born since 1990 are likely to be net contributors as well (*cf.* Verbon et al. (1995)). These accounts suggest that some generations are more successful than other generations in redistributing resources towards them. This raises the question what causes this outcome?

The decisions on the size and composition of these redistributive policies are made in the political arena. It is there where proponents and opponents of certain policy measures meet and decide what policy to enact. In most countries, these decisions are taken in representative democracies by politicians who are elected to represent the interests of their voters. Besides, interest groups may exert pressure on the politicians to enact the policy most preferred by that interest group. Therefore, the policies chosen often are a political compromise between the groups involved. Focusing on the intergenerational aspects of redistributive policies, these groups are the different generations involved. Given that these policies are the result of a political conflict between the generations involved, the question then is: How can this conflict be modelled, what are the determining factors and how do they affect the outcome of this conflict? This is the central question of this thesis and, therefore, the analysis performed in this thesis can be labelled as positive.

Thus, focusing on intergenerational conflict and redistribution, what are the factors explaining the outcome of this conflict? Besides traditional explanatory factors like

[1] Norway, Germany, Italy and Sweden. OECD (1995).

interest rates or rates of time preference, the age structure of the population also forms an important factor in explaining the size and composition of these redistributive policies. For example, a relatively large number of elderly can give them more political influence because, as a group, they possess a relatively large number of votes and, depending on their degree of organization in political pressure groups, lobbying and similar activities will be more successful. As a result, redistribution from young to elderly might be larger. Moreover, these explanatory factors are not constant over time. *E.g.* real long-term interest rates varied between 3% and 12% between 1960 and 1992. And also the age structure of the population changes over time. Table 1.1 provides data on the development of the dependency ratios, *i.e.* the number of elderly relative to the number of young, for some major industrial countries. From this table, an increase in the dependency ratios can be observed. From between 0.17 and 0.25 in 1950, they now range between 0.27 and 0.34 and are expected to increase further in the future. In 2025 they are expected to range between 0.37 (Spain) and 0.59 (Germany). In many countries, de-

	1950	1960	1970	1980	1990	2000	2010	2025
Austria	0.249	0.301	0.363	0.318	0.323	0.332	0.374	0.495
Belgium	0.254	0.299	0.329	0.298	0.322	0.339	0.367	0.489
Canada	0.192	0.192	0.195	0.215	0.247	0.258	0.315	0.460
Denmark	0.222	0.261	0.298	0.327	0.322	0.323	0.408	0.529
France	0.265	0.296	0.317	0.284	0.298	0.315	0.353	0.460
Germany[a]	0.223	0.263	0.335	0.309	0.323	0.399	0.440	0.589
Italy	0.199	0.223	0.262	0.285	0.318	0.364	0.398	0.477
Netherlands	0.194	0.230	0.249	0.253	0.265	0.287	0.368	0.549
Norway	0.225	0.280	0.317	0.352	0.347	0.312	0.362	0.484
Spain	0.176	0.204	0.246	0.254	0.277	0.308	0.311	0.370
Sweden	0.242	0.285	0.329	0.374	0.377	0.360	0.446	0.528
United Kingdom	0.250	0.282	0.327	0.341	0.343	0.334	0.369	0.461
United States	0.199	0.237	0.243	0.254	0.268	0.257	0.293	0.420

a: The figures for Germany are for former West-Germany

Source: United Nations (1992)

Table 1.1: 60+/15-59 ratio's since 1950

pendency ratios in 2025 will be at least 1.5 times as high as they are in 1990. In Germany almost twice as high and in the Netherlands even more than twice as high. Hence, the economic and political situation changes continuously. This puts redistributive policies under constant political pressure to change as well.

The redistributive policies considered in this thesis are public debt, public pensions and public investment. These types of policies capture the effects of most other types of redistributive policies as well. *E.g.* public health care redistributes in a similar way as public pensions, namely from young to elderly. Expenditure on public education has similar redistributive effects as investment in infrastructure. Next, a short introduction into these types of policies will be given. The focus is on the intergenerational conflict over these policies. Besides, some historical background as well as some normative aspects will be considered.

1.2 Public debt,......

Of the many topics economists have worried about, both theoretically and empirically, there are a few which had constant attention. One of these is public debt. Public debt redistributes resources from the future to the present[2]. For current generations, issuing debt implies more resources available because taxes decrease or public expenditures rise. Issuing debt also implies that at some future date, taxes have to increase or expenditures have to be cut in order to redeem this debt, including the interest obligations since foreign and domestic lenders will prevent a government from continuously issuing debt without repaying anything of the existing debt or its interest obligations. Hence, the generations then alive see their spending possibilities decrease. Conflicts of interest arise if some part of the current population, notably the current older generations, reasonably expects to avoid these future tax repayments and therefore prefer debt issuance. Younger generations may expect that the tax burden associated with debt redemption will fall onto their shoulders anyhow, either now, or some day in the future. Younger generations may prefer, therefore, less debt to avoid a large increase in future taxes since they want to smooth consumption over their lifetime. Besides, the current demographic development implies a decrease in the future tax base. In anticipation on this development, current

[2]This assumes that Ricardian Equivalence does not hold. Cf. footnote 6 and Chapter 2.

younger generations may even prefer a lower level of debt.

How have scholars in the past dealt with politics and public debt? In the writings of classical economists like Smith, Ricardo or Mill, great concern was expressed about the use of public debt. Especially Smith[3] and Ricardo[4] were strongly opposed to the use of debt. Both held the opinion that even in times of large public expenditures (*e.g.* war), these expenditures should be paid out of tax revenues or revenues from government property. In their view, myopia[5] on the side of the politicians and the general public would lead to profuse public expenditures and should therefore be avoided[6]. Mill[7] was less strongly opposed and noted that if taxes were too distortionary this might be a reason to use debt. Also the fact that future generations of tax payers could benefit from current expenditures could be a reason to use debt since he considered it not unreasonable to confront these future generations with a part of these expenditures. However, also Mill warned for the misuse of public debt. This classical view on public debt was dominant in the political discussion until World War II and was reflected in the norms on the budget adopted by many countries. The strongest of these was the requirement to balance the budget. This was the prevailing norm in most countries in the 19th century, though it was not always strictly obeyed. In the early 20th century, less restrictive norms were used allowing the government to borrow, to a certain extent, for public investment. Then, after World War II, due to Keynes, things changed dramatically. For Keynes' demand-side economics, public debt was an instrument to stimulate aggregate demand in times of recession, to be redeemed in economically better times. In the Keynesian view, not all economic resources are fully employed because individuals may be myopic or liquidity constrained. This causes current consumption to be very sensitive to changes in current disposable income. Hence, a debt-financed decrease in taxes will increase con-

[3]Smith (1776,1976), Book V, Chapter III, pp. 441-486.

[4]Ricardo(1817,1951), Chapter XVII and Ricardo (1820,1951)

[5]Myopia here means: not taking long-term consequences into account. In the upcoming chapters, the word 'myopia' or 'myopic' is also used but with a slightly different meaning. In the context of this thesis it means: not taking the utility of future generations into account.

[6]However, Ricardo noted an equivalence between debt and taxes, which has become known as Ricardian Equivalence, since the gift of a bond to a citizen does not make the citizen richer or poorer, because the value of the bond is offset by the value of the future tax liability. See Chapter 2 for further details on this.

[7]Mill (1848,1985), Books IV and V

sumption and, hence, investment and national output. Thus, the economy is lifted to a higher level of welfare. These theoretical prescripts, developed in the 1930s, became popular in the political debate in the 1950s and beyond[8]. Many countries tried to cope with the economic crisis of the 1970s and 1980s by demand stimulating debt-financed policies. Debts and deficits have increased hugely since then and are generally regarded as too high. Some have argued[9] that these large debts prove the failure of Keynesian policy. The reason for this failure is that politicians did not act as farsighted philosopher kings but acted myopically, exactly the behaviour the classical economists warned for.

Thus, the behaviour of politicians is important for understanding the evolution of public debt. The classical economists held the view that politicians were myopic, in Keynes' view politicians were farsighted and acted like philosopher kings. Today, there is a variety of views on behaviour of politicians. They may pursue their own private interests or the interests of the group they represent and these interests need not coincide with the 'general' interest. Besides, they may be subjected to all kinds of political pressures. Most existing studies on politics and public debt, however, deal with direct democracies where the median voter is decisive, *e.g.* Cukierman and Meltzer (1989), Tabellini and Alesina (1990), Tabellini (1991). Thus, the voters directly decide on a debt policy, not the politicians. In Cukierman and Meltzer (1989), voters differ in their human and non-human capital endowments. The current median voter may prefer a positive amount of public debt because his children are wealthier than he is and altruism towards his children is weak. Then, public debt is an alternative way to leave a negative bequest. In Tabellini and Alesina (1990), voters have different preferences on the composition of government expenditure. The current median voter uses debt to tie a future median voter, with different preferences, to a certain policy. Tabellini (1991) deals with the possibility of debt repudiation which redistributes from elderly to young and from rich to poor. In his two-period model, voters in the first period differ in their income. In other studies, public debt is used strategically by a government to tie a future government with different policy preferences, to a policy preferred by the current government (Persson and Svensson (1989), Alesina and Tabellini 1990)). For a more extensive review of these studies, the reader is referred to the introduction to Chapter 3. The model in

[8]The first country to adopt a Keynesian policy norm was the U.S. in 1947 (Stevers (1993)).

[9]See *e.g.* Buchanan and Wagner (1977).

Chapter 3 differs from the literature mentioned above because it uses a representative democracy setting since this is the political system in most western countries. Each period, the government, formed by the representatives of the generations alive, chooses a level of debt which optimizes the welfare of the generations where each generation is weighed with its (relative) political importance. Since they will not be confronted with the repayments, present old generations prefer a maximum level of debt. This maximum is determined by foreign and domestic lenders. Because of this maximum level, the present young may expect a large increase in future taxes if debt is increased to its maximum level. Because of preference for consumption smoothing, they prefer a smaller level of debt or maybe even a surplus. The evolution of public debt, due to this conflict, is studied. Besides, the effects on this evolution due to changes in the interest rate, the population growth rate, the individual rate of time preference and the political weight are analysed.

1.3 ......public pensions.....

In many countries, private and public pension schemes exist side by side. Whereas private pension schemes are usually funded according to a capital reserve (CR) scheme, *i.e.* each individual saves for its own retirement, many public pension schemes are financed using a Pay-As-You-Go (PAYG) scheme. *I.e.* current pension payments are financed by current contributions. Thus, PAYG pension schemes redistribute from the current young to the current elderly. The intergenerational conflict is clear. In the absence of altruism, the elderly favour a pension transfer from the young whereas the young generations prefer not to give a pension at all, even if they anticipate to receive a pension transfer themselves. They are not bound by an (implicit) social contract, so that their pension depends on the pension transfer they give to the current elderly. In absence of altruism or social contracts, the political power of the elderly prevents the current young from completely bleeding the elderly dry.

The development of the dependency ratios, given in table 1.1, may put severe political pressure on the current size of the PAYG scheme. If current levels (in terms of welfare) of public pensions are to be maintained, this implies that pension premiums would have to increase substantially. For countries like Germany and the Netherlands they would even have to become twice as high. Alternatively, keeping pension premiums at their current level would imply a considerable decrease in pensions for the future elderly. Both options are extreme and therefore not realistic. A political compromise with lower pensions, together with higher contribution rates, seems more likely in the future though conversion into a capital reserve system has been discussed as well. However, pension reform is a touchy subject in politics because, in general, it is not Pareto improving. An illustration of this were the proposals for pension reform in the Netherlands in 1994 by the Christian-Democrats. These proposals were met with severe opposition from the elderly and they were forced to retract them. In the elections shortly afterward, the elderly, organized in two political parties, even managed to get sufficient votes for a number of seats in parliament. Not surprisingly, most of these votes came from individuals who used to vote for the Christian-Democrats.

The example above illustrates that state provided public pensions are regarded as a natural part of the modern welfare state. However, the existence of public pensions is only a quite recent development. The first public pension scheme was enacted in Germany in 1889[10]. Since then many countries followed. For social security systems to be feasible, a certain basic level of development is necessary. Hence, it was only after the industrial revolution that many countries started to develop social security programs. But, as argued by Verbon (1988), it was also the case that due to the industrial revolution, traditional old age care was broken down. Before the industrial revolution people lived mainly in small communities with strong social ties. These personal ties guaranteed some form of care for the elderly. Due to the industrial revolution, these ties were broken which created a need for a more general social security program. However, this, in itself, was no guarantee that social security systems were indeed installed. In the early days of the industrial revolution some social security was provided through small private insurance schemes (England) or guilds (Germany). But, towards the end of the 19th century larger state intervened social security programs were installed. Several reasons are put forward for this development. First, the working class became increasingly organized in that period, which on the one hand led to increased political influence of the working class, positively affecting decisions on public pensions. On the other hand, it is argued

[10]See Verbon (1988) for an extensive description of the history of pension schemes in Germany, the Netherlands and the United States. Some basic facts concerning social security systems are also given in Sala-i-Martin (1994).

that social security systems were enacted to take the wind out of the working class' sails and prevent social unrest. Second, social security increased labor productivity and, due to the economic development, it could now be afforded. Third, politicians might depend on the working class' votes. As noted by Verbon (1988), none of these explanatory factors is by itself sufficient or even necessary, but they are all part of the story[11]. It is clear that not only the economic development but also changes in the political situation play an important role in the emergence of social security systems.

Until World War II, most social security systems were financed on a capital reserve basis. Since then, many countries switched to a Pay-As-You-Go financed system for a variety of reasons. Besides, economic development and population growth were such that a PAYG pension scheme was indeed feasible. The emergence and evolution of PAYG social security systems has been the subject in many theoretical studies. Some authors have modeled the social security system as the outcome of a voting process in a direct democracy (Browning (1975), Boadway and Wildasin (1989)). Others have used altruism as an important explanatory factor (Hansson and Stuart (1989), Veall (1986)). In Hansson and Stewart (1989) the decisions on the PAYG system are based on a unanimity rule, each present generation has veto power on proposed amendments to the system. In Veall (1986) the working generation decides. Sjoblom (1985) and Verhoeven (1993) have used the notion of a social contract to explain the existence of PAYG public pensions. In these papers, current young generations give a pension to the current elderly if they are supposed to do so according to the (implicit) social contract since, otherwise, they are 'punished' by the next period young who will not give a pension transfer to the current young when old. Again others explain the existence of PAYG social security in a representative democracy where each generation involved has political influence (Verbon (1988), Verbon and Verhoeven (1992)). This is the approach followed in Chapter 4 and it is used to study the evolution of a PAYG pension scheme.

It is not hard to guess that there is a link between debt and PAYG pensions. Both redistribute between generations though in a different way. Whereas public debt redistributes between generations over periods, PAYG public pensions redistribute between

[11] In a recent paper by Sala-i-Martin (1994), the emergence of social security systems is linked to the literature on endogenous growth. Social security is a way to get older, less productive workers out of the labor force. This increases labor productivity.

generations within periods. However, a PAYG pension can be seen as an implicit form of debt. Current young generations give a pension transfer to the current elderly and future generations repay this 'debt' by giving a pension transfer to the current young when they retire. Public debt and PAYG pensions do have different effects, however[12]. Hence, public debt and PAYG pensions can exist side by side. *E.g.* Homburg (1990) and Breyer and Straub (1993) use public debt to transform a PAYG public pension scheme to a CR pension scheme in the presence of distortionary taxes. Public debt is used to compensate current generations for the loss incurred in terms of utility because of the conversion. In Tabellini (1991), young voters, motivated by the desire to avoid intragenerational redistributions through debt repudiation (elderly differ in wealth and, thus, in the number of governments bonds they owe), may accept a transfer to the elderly even if they would have opposed such a transfer if it was voted on in isolation. The reason is that by issuing public debt, the intergenerational redistribution is tied to the intragenerational consequences of choosing how much debt to repay (Tabellini (1991), p. 354).

1.4 ......and public investment.

In case of public investment, the redistributive impact depends on the type of investment. Public education is an example of redistribution from the current working and old generation to the youngest generations. Investment in infrastructure is redistribution from older to younger generations as well. Infrastructural investment often takes a considerable time to build and many costs are incurred long before the investment generates revenues in the form of, *e.g.*, a higher productivity of private capital. Financing investments out of current tax revenues may put a large share of the burden onto the shoulders of current older generations relative to the benefits of the investment they have, since these benefits are partly realized in future periods when they are no longer alive. Therefore, current older generations may not favour public investment but may be more interested in tax cuts or more public consumption. Current young generations may, due to the public investment, receive higher wages in the future if they are still working or, if not, the interest revenues on their old-age savings may increase. In addition, there

[12] If Ricardian Equivalence does not hold, otherwise they are essentially the same as *e.g.* demonstated in Calvo and Obstveld (1988) or Buiter and Kletzer (1990). Ricardian Equivalence is treated in more detail in Chapter 2.

may be spillovers to future generations who also receive a higher wage. This increases the future tax base which makes, *e.g.*, higher PAYG pensions possible. This is beneficial to the current young.

Until recently, economists have dealt with public investment mainly from a normative point of view. In this sense, the importance of the government with respect to investment in capital was already recognized by Adam Smith who in his Wealth of Nations called it a duty of a government *'erecting and maintaining those public institutions and public works, which, though they may be in the highest degree advantageous to a great society, are, however of such a nature, that the profit could never repay the expence to any individual or small number of individuals, and which it therefore cannot be expected that any individual or small group of individuals should erect or maintain'*[13] The types of investment Smith had in mind were defence, justice, infrastructure and education[14]. Since the days of Smith, economic thought has developed and notions like 'public goods', 'externalities', etcetera, and, more general, 'market failures', have been developed. The normative role of government in correcting for market failures is widely recognized and accepted. Besides, there is a role for government in providing the right institutional environment for markets to operate in, like a legal system protecting property rights, comparable to what Smith called 'justice'. By correcting market failures, a government can increase economic efficiency (*i.e.* Pareto efficiency)[15]. Public goods like defence[16] can best be provided by a government. Infrastructure also has characteristics of a public good and, besides, has external effects, which makes provision by the government justifiable. Education is a typical example of a good which in principle could be fully provided through the market. However, because of positive external effects and, in addition, for

[13]Smith (1776,1976), p. ii 244.

[14]Smith, however, treats defence and justice separately and calls them the first and second duty of the sovereign. This quote is about the third duty of the sovereign. However, Smith links this duty to the first two by explicitly mentioning *'public institutions and public works necessary for the defence of society, and for the administration of justice'* (Smith (1776,1976) p. ii 244) before mentioning infrastucture and education.

[15]Problems arise if the government has not sufficient instruments to attain the first-best allocation. Then it has to make a trade-off between the efficiency gain on the one hand and the efficiency loss due to the use of a distortionary instrument on the other hand.

[16]It can also be argued that defence is, like justice, also necessary for markets to operate since it *e.g.* protecs against (foreign) invaders.

reasons that go beyond economic efficiency, and have to do with equity and other ethical reasons, education is provided (to a certain extent) by governments. Today, the types of goods mentioned by Smith are still mainly provided by governments. But, besides the points raised above, which can be found in every textbook on public finance, and are treated there in more detail, public investment in these goods received little extra attention[17]. This lasted until the mid-eighties. Then three developments put public investment again at the forefront of economic analysis.

First, in 1989, a series of emprical investigations by Aschauer[18] appeared dealing with the question whether the decline in productivity in the United States and some other countries could be attributed to a shortage of public capital. A survey of these studies can be found in Gramlich (1994) or Munnell (1992). The divers results obtained make it hard to judge whether there has been a shortage of public capital in general or of some types of public capital in particular and whether this shortage persists (*cf.* Gramlich (1994)). Second, an increase in focus on investment in public capital was caused by the literature on endogenous growth. One way of generating growth endogenously is by investment in public capital, *e.g.* investment infrastucture or investment in human capital by (publicly provided) education (Barro (1990)). In a standard neo-classical production function there are diminishing returns to scale for each production factor. Hence, steady-state growth is only caused by exogenous factors like the population growth rate. Public capital increases the rate of return of private capital such that there is constant return to a broad sense of capital (*i.e.* private and public and/or human capital). As a result, the growth rate becomes endogenous (*cf.* Barro and Sala-i-Martin (1995)). Third, the increased integration of the European countries and the world-wide integration of financial markets puts public investment high on the political agenda. As fiscal policy becomes increasingly harmonized in Europe, competition between countries may shift to the provision of the best environment for firms to operate in. Needless to say that infrastructure is an important aspect of this. *E.g.* in the Dutch memorandum on the national budget 1995 ('Miljoenennota 1995') it is noted that *'high-quality infrastructure, [.....], is inextricably bounded up with the competitiveness of an economy and the ability to create employment. In the globalizing world economy structural factors determining*

[17] A notable exception is Arrow and Kurz (1970).

[18] Aschauer (1989a, 1989b)

the location of businesses are eventually decisive; the quality of infrastructure is an important consideration in that.[19]

This renewed interest also raises questions on what determines public investment. However, studies approaching public investment from a positive point of view by taking the political aspects into account are scarce. There are a few exceptions. One exception is Jappelli and Ripa di Meana (1994), which deals with public investment in a representative democracy. In their model, the government has to choose between spending on public investment or public consumption. The weight given to the elderly lowers the level of public investment. Their analysis, however, contains some serious flaws[20]. An other study is *e.g.* Konrad (1993) where the elderly solely decide on the level of public capital and on a social security tax. They choose a positive level of public capital because it raises the tax base for the social security tax. Alesina and Rodrik (1994), who focus on the relation between inequality and growth, let the decision on the level of public capital be made in a direct democracy where voters differ according to income and wealth. The voter with the median income and wealth prefers a level of public capital which is higher than the level that maximizes economic growth. Public capital is financed by a tax on capital. Because the median voter, who is endowed with some labour income, prefers a higher tax rate on capital than the growth maximizing tax rate, there is overinvestment in public capital. In Van der Ploeg and Van de Klundert (1991), short-sighted politicians, who have a higher rate of time preference than the private sector, increase public consumption at the expense of public investment. This gives lower economic growth. In the introduction to Chapter 5, these studies are reviewed in more detail.

The model used in Chapter 5 focuses on the intergenerational conflict over the level of public investment. While current old generations do not benefit from public investment, current young generations benefit from public investment in the form of a higher future return on savings. Besides, investment in public capital generates spillovers to future generations in the form of higher wages. In a Barro (1990) fashion, public capital is included in the private production function[21]. Three sets of policy instruments are

[19] Miljoenennota 1995, p. 33 (original text in Dutch).

[20] These are discussed in more detail in Chapter 5.

[21] One could also imagine that it enters the private utility functions. *E.g.* a better road increases the utility of the individuals using it. This approach is, however, not followed here.

analysed. Two of them are balanced budget policies. In the first, the government can tax both generations at an uniform level only. In the other, the government is able to tax generations differently. The third set of policy instruments reflects that some countries have policies or policy proposals allowing the government to use debt for investment expenditures to a certain extent. In Germany, the constitution explicitly limits the government in using debt. The increase of public debt may never exceed the increase of investment in public capital (Fehr and Gottfried (1993)). In the Netherlands, before World War II, debt finance was only allowed for public investment (Stevers (1993)). Reestablishment of this norm has been discussed. It is included in the coalition agreement of the current government[22].

1.5 Outline of the rest of this thesis

The focus of this thesis is: intergenerational redistribution and conflicts between generations. In the following chapters, redistribution between elderly and young, between current and future generations, is analysed in a representative democracy setting. *I.e.* the government only takes the utilities of the current living generations into account. When choosing its policy, the government weighs each generation according to its relative political influence. Chapter 2 analyses, in some detail, how the intergenerational conflict is modelled in this thesis. What type of model to use, which behavioural assumptions to make, how others dealt with the intergenerational conflict, etcetera.

Chapter 3, which is based upon Meijdam, Van de Ven and Verbon (1995), develops a positive model of political decision making on public debt in a representative democracy. Non-altruistic older generations favour public debt since it enables them to shift the burden of taxation into the future. Younger generations may be more interested in smoothing the burden of taxation over their lifetime. Since future policies are relevant for current generations, the government does take the effect of the current debt policy on future policy choices into account. An analytical solution is derived for the time path of debt and taxes. Decreasing as well as increasing debt levels can be obtained. Conditions are given determining which of these patterns prevails. Also the effects of anticipated

[22]However, these norms have received criticisms as well since short-sighted politicians will try to mark certain consumption expenditures as investment expenditures so that they are able to use debt finance.

and unanticipated shocks in the exogenous parameters on the time path are analysed. The chapter concludes with a summary and a comparison with actual debt policy after World War II.

Chapter 4, which is based upon Van de Ven (1995a), analyses the evolution of a PAYG pension scheme in a representative democracy. Non-altruistic elderly favour a pension transfer from the current young generation. The willingness of the current young generation to participate in a PAYG pension system depends on the pension they expect to receive themselves when they retire. Therefore, as in Chapter 3, the effect of current policy choices on future policy choices is taken into account. The size of the PAYG pension system depends on the relative political weights of the present generation and not on some social contract. The resulting policy is compared to the policy chosen if future policy is taken as given. A comparison is also made to the policy choice of a government maximizing a social welfare function where also the utility of future generations is included. Also, the welfare effects of the introduction of a PAYG scheme in a representative democracy setting for the current generations is analysed. Furthermore, changes in the exogenous parameters, like the population growth rate and the interest rate, are analysed.

Chapter 5, based upon Van de Ven (1995b), studies investment in public capital in a similar representative democracy setting as used in Chapters 3 and 4. The chapter starts with a discussion of the definition of a command solution in a representative democracy. Next, the decentralized solution is derived. Public capital has several effects on the economy. Depending on the substitutability and complementarity between private and public capital, there is crowding-in or crowding-out of private capital. Public capital also has effects by the way it is financed, by taxes or by public debt. Therefore, the effects of different sets of policy instruments on the level of public capital are analysed. Compared to the command level, over- or underinvestment is possible. Two types of balanced-budget policies are studied, one where the government has to tax all existing generations uniformally, and another where the government can discriminate between different generations. The latter tax system operates as PAYG pension scheme. Furthermore, the case where the government is allowed to use debt to finance public investment is studied.

Chapter 6 concludes this thesis.

Chapter 2

Modelling intergenerational conflict

2.1 Introduction

In the previous chapter an introduction was given to the topic of this thesis, intergenerational conflict and intergenerational redistribution. Generations have different preferences over the size and composition of public expenditures, including those expenditures that redistribute resources between generations. It is the government that decides on these expenditures and in most countries it operates in a representative democracy setting. In this setting, the government is formed by politicians who are elected to represent the interests of their voters. Besides, interest groups may exert pressure on the politicians to persuade them to enact that policy which is preferred by that interest group. The policy chosen is therefore often a political compromise between the different preferences of the groups involved. When dealing with intergenerational redistribution, the groups involved are the different generations present.

The assumption of a representative democracy has interesting consequences since it implies that each period there is a new government, formed by the representatives of the then living generations. But, policy choices of past governments affect current policy options. Moreover, since generations live longer than one period, policy choices by future governments may be relevant for (some) present generations since they will still be alive at that time. These present generations may therefore anticipate on these future policy choices when deciding on their own preferred policy today. An additional point concerns determination of a command solution, *i.e.* the policy chosen by a dictator which can directly set consumption and capital levels. If the scope for policy-making for the dictator is confined to one period then behavioural assumptions with respect to future policy choices may have to be made. If the dictator not only cares about present generations but about future generations as well, apart from the difference in instruments, there is an additional source of difference between the dictator's policy and the decentralized representative government's policy, namely, the disregarding of these future, yet unborn, generations.

This chapter deals with how to model intergenerational conflict and how to deal with the relation between current and future policy choices. For that purpose this chapter can be divided into two parts. The first part, sections 2.2 and 2.3, discusses the choice of the necessary inputs like overlapping-generations models and game theory. In the second part, section 2.4, first, other studies dealing with political conflicts are reviewed. Next, the inputs developed are used to construct a model of a government in a representative democracy as it is used in the following chapters. Besides, the relation to a social welfare maximizer and a dictator is discussed. These are both used to compare the policy choice of a representative government to. The final section discusses some specific details related to the following chapters.

2.2 Redistribution and overlapping generations

The focus of this thesis is intergenerational conflict and redistribution via public debt, public pensions and public investment. Analysing intergenerational conflict implies studying relatively long-term phenomena. Neoclassical economics, often distinguised as dealing with long-term tendencies of economies (*cf.* Bernheim (1989b)), is therefore the proper paradigm to embed the analysis in. When dealing with redistributive issues, there is one notion worth pointing out and that is the work centering around 'Ricardian Equivalence'. Ricardo (1817) considered tax finance and debt finance to be equivalent. For a given level of expenditures, financing by debt or taxes was equivalent since individual tax payers would recognize the future liabilities associated with debt and would increase their savings appropriately. As a result their consumption pattern over time is unchanged. However, Ricardo did not really believe this to hold in practice. In his view people were myopic and issuing public debt would lead to profuse expenditure. In 1974

Ricardian Equivalence was 'rediscovered' by Barro. In a series of papers following Barro (1974), the exact requirements for Ricardian Equivalence were disentangled. Among others, they include, first, rational farsighted individuals should either live forever or should through altruisticcally motivated bequests or gifts be linked to their immediate descendents and, thus, behave as if they live forever. Second, capital markets operate perfectly. Third, taxes are non-distortionary. Fourth, there are no differential borrowing rates. Fifth, there is no uncertainty with respect to future taxes and income. The implications of these assumptions went further than just the equivalence between taxes and debt. Also intergenerational redistribution via a PAYG pension scheme becomes useless. Individuals offset the redistribution by changing their private savings appropriately. In fact, how the government finances its expenditures becomes irrelevant. The Ricardian Equivalence theorem has been subject to many critical studies (*cf.* Bernheim (1987, 1989b), Barro (1989), Bernheim and Bagwell (1988), Seater (1993)). Besides inconclusive empirical evidence (*cf.* Seater (1993)), there is no dispute over the implausibility of the strong assumptions underlying the Equivalence theorem. Taxes usually are distortionary, capital markets do not operate perfectly and there is uncertainty with respect to future developments. In this thesis it is assumed that individuals are non-altruistic. Though altruism is present in reality as well (see *e.g.* Van der Heijden et al.(1995)) it is not likely to exist in the specific form formulated by Barro (1974). In reality, there are many intrafamily linkages (Bernheim and Bagwell (1988)) or impure forms of altruism due to incomplete information, to utility derived from the 'joy of giving' or to constraints on intergenerational altruism (Abel and Bernheim (1991)). This leads to the conclusion that the Ricardian view is at most an interesting intellectual bench-mark. For policy analysis, however, it is clearly unsuitable. Therefore, in this thesis, Ricardian Equivalence does not hold.

The next question is, which model to use? When dealing with intergenerational redistribution, it is not surprising that overlapping-generation (OLG) models form an important tool for the analysis in the following chapters. In an OLG model, at each point in time, a part of the current population dies and is replaced by new individuals or generations. Since Ricardian Equivalence does not hold, generations are not linked by altruism as in

Barro (1974)[1]. OLG models, in turn, can be subdivided into two subclasses. There is the 'classic' Samuelson-Diamond OLG model (Samuelson (1958), Diamond (1965)) and the 'modern' Blanchard-Yaari OLG model (Blanchard (1985), Yaari (1965), Weil (1989)). In the Samuelson-Diamond OLG model time is discrete and each period a new generation is 'born' living for a finite number of periods, usually two or three, after which they 'die'. It is not difficult to see that at each point in time the number of different generations alive is exactly equal to the number of periods each individual lives. Note also that, and this is important for the sequel, at each point in time, members of different generations have a different remaining lifetime. The second type of OLG model, the Blanchard-Yaari model, differs from the 'classic' OLG model by the fact that individuals do not have a finite number of periods to live but face, at each point in time, a constant probability of dying. The implication of this assumption is that at each point in time, each individual alive, irrespective of its date of birth, has the same expected remaining lifetime. This explains why this model is also known as a model of 'perpetual youth'. Generations differ in the amount of wealth they have accumulated over their lifetime.

An important driving force behind intergenerational conflicts is the fact that members of different generations face a different remaining lifetime over which they plan their consumption, savings etcetera. Since in the Blanchard-Yaari model, at each point in time, each individual alive has the same expected remaining lifetime, the driving force behind the conflicts studied in this thesis is absent in that model. Thus, the classic Samuelson-Diamond OLG model, where at each point in time, members of different generations face a different remaining lifetime, is the most appropriate model for the object of this study[2]. Generations will favour policies which are beneficial to them during their lifetime without incurring (part of) the costs since these are shifted onto the shoulders of other present or future generations.

In the upcoming chapters, a two-overlapping-generations structure is used. Individuals

[1]This kind of altruism would give a Ramsey type of model, because individuals would act as if they live forever and Ricardian Equivalence would hold.

[2]Differences in remaining lifetime are certainly important when dealing with redistribution via public debt or public investment. Though this does not hold for PAYG public pensions, modelling PAYG public pension schemes is much simpeler in a Samuelson-Diamond OLG model without losing any of the essential features.

live for two periods after which they die. In the first period they work and earn labour income or they receive an initial endowment. In the second period they are retired and rely for their consumption on their own private savings or some form of pension transfer from the government. This structure implies that at each point in time two generations are present. They will be called 'young' and 'elderly' respectively. Of course, the young in one period are the elderly in the next period. Furthermore, they are assumed to be non-altruistic. The individuals will be assumed to optimize some lifetime utility function U which is taken to be additive separable over time. Hence, a member of the generation born at time t optimizes

$$U_t \equiv U(c_t^y, c_{t+1}^o) = u(c_t^y) + \theta v(c_{t+1}^o), \qquad 0 < \theta < 1 \tag{2.2.1}$$

where u and v are the direct utility functions in the first period, when young, and the second period, when old, of their lives respectively. c^y denotes consumption when young, c^o denotes consumption when old. θ is the factor at which individuals discount their future consumption.

2.3 Games and dynamics

Conflicts between generations form an important aspect of this thesis. Situations of conflict can be analysed with the use of game theory which deals with situations in which several persons or institutions (players) are involved with (possibly) conflicting interests. In the present analysis, the players involved are the current and future generations and the different governments since, in each period, a new government is formed by the representatives of the generations then alive. This implies that, each period, two games are played. One between the private sector, consisting of the generations present, and the government. The other between current and future governments because current governments may have to take future policy choices into account when choosing their own policies since these future policy choices may affect the utility of current generations. This dependence of present policy choices on future policy choices and, besides, the fact that current policy options depend on past policy choices, makes this game intrinsically dynamic[5]. More precisely, the present state, *i.e.* the resulting situation due to the cur-

[5]Though not a real 'dynamic game' as will be seen below.

rently chosen actions by the players, depends on past states (which, of course, are the result of past actions). *E.g.* the size of the current PAYG pension system may depend crucially on the past savings of the current elderly. The inherited level of public debt, due to past policy choices, affects current policy options. Besides, the creation of public debt affects future policy options since this debt has to be redeemed some day in the future and a part of this future debt redemption may fall onto the shoulders of some current generations.

When applying game theory, an important assumption which has to be made concerns the behavioural attitude towards the actions of the other players[6]. There are two possibilities. First, when choosing his own action, each player takes the actions of the other players as given. This is a case of Nash behaviour. Second, there is Stackelberg behaviour in which players act in a hierarchical order and announce their actions one after the other. Players lower in the decision-hierarchy know the actions of the players higher up in the decision-hierarchy and take the actions into account when deciding on their own actions. Players higher up in the decision-hierarchy take the reactions of players lower in the decision-hierarchy on their actions into account. A player higher up in the decision-hierarchy is a Stackelberg leader with respect to the player lower in the decision-hierarchy, who is a Stackelberg follower with respect to the leader.

When players act in different periods, Stackelberg behaviour seems the most natural assumption. The players acting in earlier periods are Stackelberg leaders with respect to players acting in later periods who are the Stackelberg followers. This is the case for the government in the analysis in Chapters 3 and 4. *I.e.* the current government is Stackelberg leader with respect to the government in the next period. However, in Chapter 5 Nash behaviour towards future governments had to be assumed to keep the

[6]A second important assumption which has to be made when dealing with real dynamic games is on the available information. Two possible assumptions can, roughly, be discerned. Either, there is open loop information or there is feedback information. In the first case, there is only information on the initial state of the game. While playing the game, no information on current states becomes available. In the second case, while playing the game, each period information concerning the state at the beginning of that period becomes available. See *e.g.* De Zeeuw and Van der Ploeg (1991). But, as argued, the games analysed here are not true dynamic games. Hence, this kind of assumptions is not relevant here. An exception is when the government optimizes social welfare, *i.e.* the utilities of all current and future generations.

analysis somewhat tractable. Besides, Stackelberg behaviour can also be appropriate if one of the players is 'big' compared to the other player. This is the case for the game played each period between the government and the private sector. The government is one 'big' organization while the private sector consists of a large number of individuals, each acting atomistically, *i.e.* they consider the effect of their private actions on the actions of others negligible and, thus, take the actions of others as given. Hence, the government is Stackelberg leader with respect to the private sector.

Thus, games as well as dynamics play an important role in the analysis of the following chapters. However, though closely related, the combination of the two, the theory of dynamic games, cannot be applied. For a game to be dynamic requires, besides the dependence of the present state on past states, repeated interaction of the players. However, in the present context, players interact only once since each period the government is formed by a new set of politicians. Moreover, old generations die and new generations are born each period. On the other hand, however, past states do matter for current states. *E.g.* the inherited level of public debt is relevant for current policy choices and, thus, for the current level of debt. Past savings of the current elderly do matter for the level of the pension benefit they will receive. This implies that solutions to the games analysed here are derived in a similar way as solutions to dynamic games. However, the standard techniques, as used in dynamic game theory, cannot be applied.

2.4 Modelling political conflict

Using the materials given in Sections 2 and 3 the model of a government in a representative democracy, as used in the upcoming chapters, can be constructed. When solving this model, the difference between Nash and Stackelberg behaviour with respect to future policy choices is indicated. Also some issues in relation to social welfare maximizers and command solutions are treated. But, first, a short overview is given of other literature dealing with the politics of intergenerational redistribution.

Related literature How have others dealt with the politics of intergenerational redistribution? Most of this literature has recently been reviewed in Breyer (1994) and has also been discussed in Chapter 1. But in the previous chapter, the focus was on the political conflict itself, here, limited to dynamic models, the focus is on the modelling of the political conflict.

Most contributions are in the field of public pensions. As argued, Stackelberg leadership of current governments with respect to future governments is a natural assumption. However, Stackelberg games are often difficult to solve analytically. The existing literature has dealt with the effects of future policies on current policies in a variety of ways. Verbon and Verhoeven (1992), in a paper on PAYG pensions, assume that current policymakers take future decisions as given. Thus, Nash behaviour is assumed. In another paper on pensions, Veall (1986) makes the (inconsistent) assumption of present generations displaying Stackelberg behaviour towards future generations, while future generations display Nash behaviour towards their succeeding generations. The inconsistency arises because the generations of the next period are assumed to behave in the same way, *i.e.* display Stackelberg behaviour towards their successors and assume future generations to display Nash behaviour. In Browning (1975) and Boadway and Wildasin (1989), current generations decide on a pension policy based on the assumption that this policy is not changed in the future. In Browning (1975), this turns out to be correct, since future generations make the same assumption. In Boadway and Wildasin (1989) this leads to an initial overshooting of the steady-state values of the pension transfers after which, depending on the parameters, they gradually converge to the steady state by a pattern of alternative over- and undershooting, or they do not converge but cycle around the steady state. Hansson and Stuart (1989) assume that the chosen pension policy is laid down in the constitution. Since amendments to the chosen policy are only possible by unanimous consent, the policy is never changed.

In the case of public debt, taking future policy as given, *i.e.* Nash behaviour towards future governments, it is not difficult to see that, in the presence of non-altruistic individuals, debt policy would be completely determined by the maximum sustainable level of debt which is determined, among others, by the tax base. Thus, in this case Stackelberg behaviour towards future governments is important. Stackelberg behaviour has been introduced before in the models with public debt. Persson and Svensson (1989), where public debt is used strategically to tie a future government to a certain policy, and Tabellini and Alesina (1990), who analyse voting on the budget deficit, use Stackelberg behaviour in a two-period model. Alesina and Tabellini (1990), which is very similar to Persson and Svensson (1989) confine themselves to steady-state analysis. Tabellini (1991) develops a two period model where voters in the first period decide on a level of debt, taking the tax policy of the second period into account. In this model, altruism plays an important role as well. Altruism is also important in Cukierman and Meltzer (1989), where public debt is used to transfer resources from the future to the present because (some) children are wealthier than their parents. These studies are reviewed in more detail in the introduction to Chapter 3.

The government in a representative democracy. As mentioned on several occasions before in this thesis, the government is assumed to operate in a representative democracy setting since this is the dominant political system in most countries. The government is formed by politicians who represent the interests of various groups present in society, like workers, pensioners, employers, etcetera. Each of these groups' interests, which can be described by the utility of a reperesentative member of the group, receives a weight, called the political power, in the objective function maximized by the government. This political weight is the result of, *e.g.*, competition among the different interest groups for political influence, lobbying and similar activities by these interest groups or competition among politicians for the votes of these interest groups. Concentrating on the intergenerational conflict, it is convenient to distinguish two groups, namely young and elderly. Thus, it is assumed that the government maximizes a weighed combination of the utility of the elderly and young. In general, this political weight will be a function of the size of the generation. In the sequel this function is assumed to be linear. In that case the objective function of the government in period t can be represented by[7]:

$$U_{t-1} + \lambda(1+n)U_t = \theta v(c_t^o) + \lambda(1+n)\left[u(c_t^y) \; + \; \theta v(c_{t+1}^o)\right] \tag{2.4.1}$$

where λ is the political importance of a current young individual relative to a current old individual. n is the population growth rate. Hence, the relative political power of the young generation equals $\lambda(1+n)$. The expression after the equality sign follows from using eq. (2.2.1) where the current elderly are only included with their old-age utility. The government maximizes eq. (2.4.1) using the available instruments, I_t and is

[7]Bernheim (1989a) calls a decision function as eq. (2.4.1) *'within an overlapping generations framework, ...the most natural class of welfare functions for a representative government'* (Bernheim (1989a), p. 124.)

subjected to a budget constraint. The approach to government behaviour followed here is closely related to the interest function approach as developed by Van Winden (1983) or the model of competition among pressure groups as developed by Becker (1983). For a behavioural underpinning the reader is referred to Coughlin et al. (1990) which is based on probabilistic voting models of vote-maximizing politicians. The political weights can also be thought of as being the result of different interest groups engaging in lobbying activities in order to obtain influence on political decisions (Potters and Van Winden (1993)). An alternative interpretation of eq. (2.4.1) is that of a generation of young who decide on which policy to implement and who are altruistic towards the elderly. $^1/_\lambda$ then measures the degree of altruism.

As a result of the actions of the private and public sector, the state the economy is in evolves over time. In the notation of this chapter, this can be denoted by

$$S_t = BC(I_t, s_t; S_{t-1}; \ldots) \tag{2.4.2}$$

S_t denotes the state variable at the end of period t (and thus, at the beginning of period $t+1$)[8]. The state variable can be the level of public debt, the level of capital, etcetera. The '...' stands, as before, for other relevant variables (interest rates, etcetera) which are, for convenience, taken as given in this section. These state variables are subjected to terminal conditions, $S_N = \bar{S}$[9].

If the government in period t is a Stackelberg leader with respect to future governments, it takes the behavioural response of these future governments on the current policy choice into account. Denote the optimal policy choice of the next period government by I^*_{t+1}. I^*_{t+1} is a function of the state variable at the beginning of period $t+1$, S_t. Of course, in this policy choice, the policy choices in period $t+2$ etcetera, are taken into account. By using the budget constraint of the government, eq. (2.4.2), I^*_{t+1} can be written as a function of I_t and S_{t-1}. Then, the first-order condition for the government in period t is given by:[10]

[8] Not to be confused with private savings s_t.

[9] In general, the terminal condition can, of course, take any value. It can also be an interval.

[10] To avoid unnecessarily complex notation, it is assumed here that the government has only one policy instrument. Thus, the set I_t contains only one element.

$$\theta\frac{dv_t}{dc_t^o}\cdot\frac{dc_t^o}{dI_t}+\lambda(1+n)\frac{du_t}{dc_t^y}\cdot\frac{dc_t^y}{dI_t}+\lambda(1+n)\theta\frac{dv_{t+1}}{dc_{t+1}^o}\cdot\frac{dc_{t+1}^o}{dI_{t+1}^*}\cdot\frac{dI_{t+1}^*}{dI_t}=0 \qquad (2.4.3)$$

The inclusion of the derivative of private savings with respect to the instrument of the government reflects the fact that the government is Stackelberg leader with respect to the private sector. The term $\frac{dI_{t+1}^*}{dI_t}$ in the first-order condition of the government, eq. (2.4.3), reflects the Stackelberg behaviour with respect to future governments. If Nash behaviour is assumed, this term disappears. The solution to this model is found by starting in the final period N, where the optimal policy is determined by the terminal condition on the state variable, S_N. By recursively working backward, the complete solution to the model can, in principle, be derived.

The model sofar is intended to be positive, *i.e.* describing actual developments chosen by a representative government. Then two, interesting, comparisons can be made. One is comparing this outcome to a government that not only cares about current generations but also about future generations. *I.e.*, to a government maximizing social welfare. Comparing this model to the representative democracy gives the effects of myopia, *i.e.* the disregarding of future generations, on the policy choices. Alternatively, one might be interested in comparing the policy choices of a representative government to the policy choice of a dictator who has the same objective function but who is able to directly set consumption and capital levels. This comparison shows whether the representative government has sufficient instruments to attain the same allocation as the dictator. Attention is given to both these alternatives.

Social welfare versus representative democracy. For a government optimizing a social welfare function where the utilities of all current and future generations are included, the objective function is given by[11]

$$\sum_{i=0}^{N}[\rho(1+n)]^i U_{i-1}=\sum_{i=0}^{N}[\rho(1+n)]^i[\theta v(c_i^o)+\rho(1+n)u(c_i^y)] \qquad (2.4.4)$$

[11] This is a Benthamite social welfare function. If each generation is not weighed with its relative size, $(1+n)^i$, a Millian social welfare function would result.

where ρ is the social discount factor[12]. If $N \to \infty$, for the sum to be finite it has to hold that $[\rho(1+n)]^i < 1$. The solution to this problem is found by solving the equations of Bellman's principle of dynamic programming recursively[13]:

$$V_t(S_{t-1}) = \max\{\theta v(c_t^o) + \rho(1+n)u(c_t^y) + V_{t+1}(S_t)\} \quad t = 1, \ldots, N \tag{2.4.5}$$

$$V_{N+1}(S_N) = \max\{\theta v(c_N^o) + \rho(1+n)u(c_N^y)\} \tag{2.4.6}$$

V_t is called the value function and gives the maximum value of the objective function over the remainder of the planning period, in this example period t until period N, as a function of the state at the beginning of the current period. The first-order conditions are given by:

$$\theta \frac{dv_t}{dc_t^o} \cdot \frac{dc_t^o}{dI_t} + \rho(1+n)\frac{du_t}{dc_t^y} \cdot \frac{dc_t^y}{dI_t} + \rho(1+n)\frac{dV_{t+1}(S_t)}{dI_t} = 0 \quad t = 1, \ldots, N \tag{2.4.7}$$

$$\theta \frac{dv_N}{dc_N^o} \cdot \frac{dc_N^o}{dI_N} + \rho(1+n)\frac{du_N}{dc_N^y} \cdot \frac{dc_N^y}{dI_N} = 0 \tag{2.4.8}$$

For more details on this, the reader is referred to the many textbooks available on the topic of dynamic programming (*e.g.* Sargent (1987)).

Both problems, the representative democracy and the social welfare optimization, are solved in a similar manner. Starting in the final period, where the policy choice is completely determined by the terminal condition on the state variables, the complete solution is found by working recursively backward. Note the difference between the government operating in a representative democracy and the social welfare maximizer. In the former case, there is no value function. The current government is not interested in maximizing the utilities of future generations, it only is interested in future policy choices insofar

[12]In the term on the right-hand-side, the young-age utility of the first generation and the old-age utility of the last generation are left out. In the former case, this utility is enjoyed before the world started, in the latter, after the world terminated.

[13]Implicitely it is assumed that the government has information on the current state. Which is a trivial assumption. Hence, in the terminology of dynamic games, this is the feedback information structure. See footnote 6 of this chapter.

they affect current generations. Hence, in the first-order condition, eq. (2.4.3), there is an expression containing the derivative of the utility of the present young when old with respect to the policy instrument I_t. Note that the effect of I_t on v_{t+1} works through the Stackelberg term, *i.e.* the effect of the present policy I_t on the optimal policy choice of the future government (I^*_{t+1}). There is a priori no reason that both approaches lead to similar policy choices and, in general, this is not the case. On the contrary, one would expect that they would lead to different outcomes because, *e.g.*, if $\rho = \lambda$, a social welfare maximizer gives much more weight to future developments than a sequence of representative governments because it takes the utilities of all future generations into account. However, in some cases, for specific choices of the values of the exogeneous parameters, the two approaches can give identical policy choices. For the parameter choices giving this result in Chapters 3 and 4, it holds that λ is larger than ρ. This higher weight for the current young generation in a representative democracy compensates for the disregarding of future generations, which are taken into account by a social welfare maximizer.

Central planning versus representative democracy. The market solution for a representative democracy can also be compared to a command solution. However, when defining a command solution complications arise. Usually, a command solution is defined for a social welfare maximizer, choosing the consumption and capital levels maximizing the utilities of all present and future generations once-and-for-all. But, dealing with a representative democracy, this is not the appropriate thing to do because it implies not only comparing whether some set of instruments suffices to attain the command outcome but also comparing myopia to social welfare. Therefore, one should define a command solution for a myopic dictator caring about present generations only. But then, what to do with c^o_{t+1}? Since the scope for such a myopic dictator is confined to one period only (in this case period t), c^o_{t+1} is chosen by the myopic dictator in the following period (as c^o_t is chosen by the present myopic dictator). But, c^o_{t+1} gives utility to the current young generation, a generation the present myopic dictator cares about. Hence, behavioural assumptions with respect to c^o_{t+1} are necessary. As seen above, this can either be Nash or Stackelberg. Solving this problem is similar to solving the problem of the decentralized representative democracy. The difference is the set of intruments, which, in case of a command solution, contains consumption and capital levels. The first-order condition resembles eq. (2.4.3). Of course, behavioural assumptions with respect to the private sector are no longer relevant.

2.5 Final remarks

The previous section treated the basic model as it will be used in the upcoming chapters in general terms. In this final section, the differences between these chapters are given. I_t, the policy instruments of the government, differs in each of the following chapters. In Chapter 3, dealing with public debt, the policy instrument available to the government will be debt, obviously, and a uniform consumption tax levied on both generations. The choice of a consumption tax instead of an income tax or lump-sum taxes is driven by the fact that a consumption tax is analytically more tractable and, therefore allows to focus on the effects of the intergenerational conflict. Moreover, in most countries, pension savings are deductable from the income tax. Assuming this in Chapter 3 implies that an income tax is effectively a consumption tax. Similar reasons are behind the choice for lump-sum taxes in Chapters 4 and 5. In Chapter 4, where a PAYG pension scheme is analysed, a lump-sum tax is levied on the current young generations. The revenues of this tax are used to make a lump-sum pension transfer to the elderly. In this chapter, there is no public debt. Thus, each period, the revenues of the lump-sum tax are exactly equal to the transfer payments to the elderly. In Chapter 5, dealing with public investment, three sets of policy instruments are discussed. First, a uniform lump-sum tax levied on both generations. Second, a time- and age-dependent lump-sum tax. Finally, lump-sum taxes and public debt. The revenues of the taxes and, if possible, of public debt issuance, are used to finance public investment and, if necessary, debt repayments. Investment in public capital raises the interest rate in the next period which is beneficial to the current young. The return on their savings increases. The current elderly have nothing to gain from public investment. There is also a spillover to future generations because their wage increases.

Chapters 3 and 4 are similar in many respects. In both chapters the government is assumed to be a Stackelberg leader with respect to future governments. Besides, the policy choices of a representative government are compared to the policy choices of a social welfare maximizer. With this comparison, the effects of disregarding the utility of future generations is analysed. The decentralized solution and the command solution for a social welfare maximizer are identical. In this case the government has sufficient instruments to replicate the command solution through the market. The main emphasis in these chapters is on the evolution of government policy. In Chapter 3 this is a debt policy whereas in Chapter 4, a Pay-As-You-Go public pension policy is the object of study. To prevent the solution from being dominated by 'end-of-time' effects[14], an infinite horizon model is analysed in these chapters. Finally, both chapters deal with small open economies. Chapter 5, dealing with investment in public capital, is somewhat different. The economy is assumed to be closed. Public capital affects wages and the interest rate with a lag of one period. Thus, increases in the level of public capital is beneficial to the younger generations only because the return on their savings increases. A small open economy would imply a fixed world interest rate rendering an uninteresting problem. In that case, public investment only affects wages. Since investments in public capital become productive with a lag of one period and none of the present generations work in the following period, present generations are not interested in public investment. Closing the economy, however, considerably complicates the analysis. Therefore, the assumption of Stackelberg behaviour towards future governments is replaced by the assumption of Nash behaviour to keep the analysis somewhat tractable. However, in contrast to Chapter 3, this implies no loss of any of the essential features in this chapter. Furthermore, the analysis has to be confined to a steady-state analysis. As a benchmark case, the command solution for a representative democracy is used. Thus, the focus is on whether the government in the decentralized economy has sufficient instruments to obtain the command outcome.

[14]Imagine a two-period model. Then, if public debt has to be redeemed the second period, policy in this final period is to a large extent determined beforehand. Besides, in a two-period-model, each individuals horizon is equivalent and there is no inherent tendency to shift taxes to future periods as these future taxes have to be paid by the individuals themselves.

Chapter 3

The dynamics of public debt

3.1 Introduction

In many western countries a similar evolution of public debt after World War II occurred. First, debt ratios declined, until, somewhere in the mid seventies, they started to rise. Recently, the EC countries have proclaimed by way of the Maastricht Treaty that stabilization of debt ratios (at a relatively low value) is the policy goal to be targeted. These phenomena raise the question as to the determining factors of the evolution of debt. The age structure of the population may constitute an important factor. As already sketched in Chapter 1, older generations of tax payers may favor a debt-financed tax cut since it enables them to avoid tax payments by shifting these taxes to future periods when they are no longer alive. Younger generations, not expecting to avoid these tax payments, may favor policies which smooths these tax payments over their lifecycle. Trivially, the effect of the age structure depends to a large extent on the ruling tax regime. In the overlapping-generations model, to be developed in Section 3.2, all consumers, both young and old, pay a consumption tax at the same rate. For the elderly, tax shifting by debt creation would then be advantageous. For the young, however, tax shifting implies that they are confronted with a higher future tax burden. Though the model used in this chapter has an infinite horizon, continuously passing on debt by issuing new debt without repaying at least part of the principal or the interest burden is not possible. The domestic and foreign lenders (an open economy is assumed) will prevent the government from issuing more debt than it is ever able to repay, including the interest payments. Since it is never able to repay more than it ever can collect in taxes, this constitutes an upper limit for the level of debt which equals the present value of all future tax payments minus the present value of any future government expenditures. Besides, the government is assumed to operate in a representative democracy setting and as such optimizes the utilities of current generations only. Hence, the desire of the young to smooth taxes over their lifecycle will prevent the government from accumulating debt as fast as possible.

In such a model, ageing, *i.e.* an increase in the number of elderly relative to the number of young, has two effects. The political influence of the elderly, who prefer tax shifting, increases due to an increase in their relative number. On the other hand, a decrease in population growth leads to a decrease of the tax base. This implies that the possibilities for tax shifting decrease in the long run. As a consequence, the current young may be confronted with a higher future tax burden if taxes are shifted to the future now. Therefore, they will be more strongly opposed to tax shifting.

From this perspective, this chapter analyses the choice of the debt policy of a rational government. It is assumed that the government only takes the utility of current generations into account. So, it may be interpreted as a representative government which consists of subsequent generations of politicians with a finite time horizon. There is no way a present politician is able to bind his successors to a planned policy. This does not imply that future policy is exogenously given to a present politician: public debt is the instrument through which he is able to influence future decisions. Therefore, rational expectations of future decisions have to be formed. In particular, generations of politicians are assumed to be able to calculate all possible paths of future taxes (and, hence, debt levels) as a function of the present tax rate and to pick the path that maximizes their welfare, *i.e.* they are assumed to be Stackelberg leaders towards future generations of politicians.

The assumption of Stackelberg leadership has been introduced before in this context. Persson and Svensson (1989) and Tabellini and Alesina (1990) use Stackelberg behaviour in a two-period model. In Persson and Svensson (1989), a government with a relatively low preference for public consumption, which is sure to be replaced the next period by a government with a relatively high preference for public consumption, will issue debt in order to constrain the next-period government in its policy choice. It has to redeem the debt and, therefore, has less possibilities for public consumption expenditures. Tabellini and Alesina (1990) construct a model where heterogeneous individuals have to vote on the budget deficit. The heterogeneity stems from different preferences with respect to the composition of public expenditures. Since the median voter in both periods may differ, the median voter in the first period takes the effect of its choice on the choice of the next period median voter into account. Alesina and Tabellini (1990), which is closely related to Persson and Svensson (1989), do consider an infinite-horizon model but confine themselves to a steady-state analysis. The difference with Persson and Svensson (1989) is that a current government is not sure to be replaced by another government but faces a certain (exogenously) given probability to be replaced. Consumers are heterogeneous in a similar way as in Tabellini and Alesina (1990). As in Persson and Svensson (1989) there are different policymakers who have different preferences with respect to the size and composition of public expenditures. Again public debt is used strategically to affect future policy choices. Note that in these models each individuals horizon is equivalent to the horizon of the government. As a result, there is no inherent tendency to shift taxes to future periods as these future taxes have to be paid by the individuals themselves.

In this chapter, the difference in time horizon of different generations is at the heart of the analysis. Like the literature cited above, repudiation[1] is abstracted from. Contrary to this work, successive generations of policymakers are assumed to have identical preferences. Moreover, an infinite-time horizon is considered. This chapter also abstracts from altruism. Assuming that the policy adopted reflects the interests of the different groups present in that period, an explicit solution for the time path of public debt and the tax rate is derived. It appears that, depending on the relation between the interest rate, the population growth rate, the private discount rate and the relative political importance of the generations, public debt can decrease or increase in the course of time. In the latter case, it asymptotically approaches a finite maximum sustainable value,

[1]Tabellini (1991) studies debt as an instrument of intergenerational redistribution in a two-period model that allows for repudiation. Altruism is essential for his results. Without altruism the government budget is always in balance because rational lenders will foresee future repudiation and, therefore, will not lend to the government. If, however, they are altruistic towards future generations, they will accept a partial repudiation since this is beneficial to future generations. Other papers dealing with intergenerational redistribution in a political-economic model with altruism include Cukierman and Meltzer (1989) and Hansson and Stuart (1989).

determined by the present value of the total future tax collections minus public expenditures. Note that this result contradicts the claim made by Tabellini and Alesina (1990), who state that '*[i]n an overlapping generations model with no altruism,..., current voters would be unanimously in favor of the largest possible budget deficit,...*'[2] Tabellini and Alesina (1990) forget that current young generations prefer tax smoothing to a certain extent since they are present the next period and will otherwise be confronted with an enormous increase in tax payments.

It can be shown that the time path chosen by representative governments may, but need not, coincide with the one that results in the case of a social-welfare-maximizing government, *i.e.* a government that takes the utilities of all current and future generations into account.

The rest of this chapter is organized as follows. The next section introduces the overlapping-generations model. Section 3.3 presents the solution to the model. The ability to calculate time paths for debt also makes it possible to analyse how the course of these paths will change due to exogenous changes in the parameters. This is done in Section 3.4. Besides, a comparison is made with actual debt policy after World War II. The final section concludes.

3.2 The Model

In the sequel, a small open economy will be assumed where debtors are able to borrow from domestic lenders as well as foreign lenders, both against a fixed world interest rate. In this case, the capacity of the domestic capital market does not impose any restrictions on the borrowing behaviour of the government. This in contrast with a closed economy, where government borrowing is restricted by the domestic supply of savings.

The Consumers

A standard overlapping-generations model is used where two non-altruistic generations, old and young, are present at the same time. Each individual lives for two periods. In the first period, he is endowed with one unit of income of which he saves an amount s.

[2]Tabellini and Alesina (1990), p. 38.

He has to pay a tax rate τ over the remaining part. The rest is used for consumption in the first period of his life, c_t^y, which therefore equals:

$$c_t^y = (1 - \tau_t)(1 - s_t). \tag{3.2.1}$$

When he is old, the savings including the interest revenues net of taxes are consumed:

$$c_{t+1}^o = (1 - \tau_{t+1})(1 + r)s_t \tag{3.2.2}$$

where c_{t+1}^o is old-age consumption at time $t + 1$ and r is the given fixed world interest rate. So, at a given time the young and the elderly pay the same tax rate which excludes direct transfers from the current young to the current elderly. Note that the tax system used is actually a consumption tax instead of an income tax. s_t are savings for future consumption so that these can be considered as pension premiums which are tax deductible in many countries. The value of savings, including accrued interest, then equals the pension benefit which is taxable. The young optimize a lifetime-utility function $U_t = U(c_t^y, c_{t+1}^o)$ subject to eqs. (3.2.1) and (3.2.2). The instrument in the optimization is the savings rate. Hence, the first-order condition reads:

$$\frac{\partial U_t}{\partial s_t} = -\frac{\partial U_t}{\partial c_t^y}(1 - \tau_t) + \frac{\partial U_t}{\partial c_{t+1}^o}(1 + r)(1 - \tau_{t+1}) = 0 \tag{3.2.3}$$

From this follows $s_t^* = s_t^*(\tau_t,\ \tau_{t+1}, r)$ as the optimal amount of savings.

The Government

Government outlays at time t consist of interest payments on the given amount of government debt per consumer of generation $t - 1$, b_{t-1}, and given expenditures per consumer of generation t, g_t. These expenditures give no utility to the consumers and can be seen as an (exogenous) collection costs of taxes, independent of the amount of taxes. The government generates revenues by levying taxes. Abstracting from the possibility of debt repudiation, directly or indirectly through monetary finance, the government's budget identity reads:

$$b_t = \left(\frac{1 + r}{1 + n}\right) b_{t-1} + g_t - T_t \tag{3.2.4}$$

where n is the growth rate of the population and T_t is total tax revenue per consumer of generation t. From eqs. (3.2.1) and (3.2.2) it follows that total tax revenue equals:

$$T_t = \tau_t[(1 - s_t) + \left(\frac{1+r}{1+n}\right) s_{t-1}] \tag{3.2.5}$$

Two natural conditions constrain the set of fiscal policies open to the government. First, the government is forced by the foreign and domestic lenders to obey the well-known No-Ponzi-Game condition which simply says that the principal and the service of the debt cannot be financed completely by going into new debt. In other words, it requires that the present value of outstanding debt converges towards 0 asymptotically, *i.e.*:

$$\lim_{\mathcal{T}\to\infty} \left(\frac{1+r}{1+n}\right)^{-\mathcal{T}} b_{\mathcal{T}} = 0 \tag{3.2.6}$$

Aggregating forward eq. (3.2.4) from period t on to a final period $\mathcal{T}$, the government's intertemporal budget constraint results:

$$b_t = \left(\frac{1+r}{1+n}\right)^{t-\mathcal{T}} b_{\mathcal{T}} + \sum_{j=t+1}^{\mathcal{T}} \left(\frac{1+r}{1+n}\right)^{t-j} (T_j - g_j) \tag{3.2.7}$$

The NPG condition then implies that the intertemporal budget constraint for an infinite horizon can be written as:

$$b_t = \sum_{j=t+1}^{\infty} \left(\frac{1+r}{1+n}\right)^{t-j} (T_j - g_j) \tag{3.2.8}$$

which is the well-known result that the present debt has to be met by future primary surpluses. This condition is tightened by a second natural constraint. When raising taxes, the government faces a maximum tax rate, τ^{max}, which, obviously, never exceeds 1[3]. This implies that the total of future primary surpluses is finite (assuming that the tax base grows at a rate smaller than $\frac{1+r}{1+n}$) and thus constitutes a threshold value, b^{max}, above which the (foreign) lenders will no longer be willing to buy government debt since then they know for sure that it will not be repaid.

The threshold value b^{max} and the maximum tax rate τ^{max} constrain the set of fiscal

[3]Because of a Laffer curve effect, τ^{max} may be smaller than 1. However, here it is assumed that τ^{max} equals 1.

policies open to the government. Which policy will be chosen by the government from this set depends on the decision making process. Assuming a representative democracy setting, the government at time t represents currently living generations only and is succeeded by another government every next period. It is assumed to maximize a decision function consisting of the utilities of the current generations only:

$$W_t = U_{t-1} + \lambda(1+n)U_t \tag{3.2.9}$$

where $\lambda \in [0, \infty)$ denotes the relative political influence of a young individual. Every t, this decision function is maximized again by a new generation of politicians. In choosing its policy the government explicitly takes account of s_t^*, the optimal amount of savings chosen by the consumers. This makes it act as a Stackelberg leader towards the private sector.

Given the fact that current generations are not altruistic towards future generations, the current government does not take the utility of future generations into account. Therefore, the scope for fiscal policy for an incumbent government is confined to the present period. The chosen policy has, however, implications for the actions to be taken by future governments. It is assumed that the current government takes these actions into account in choosing a policy. This means that it also acts as a Stackelberg leader towards all future governments.

3.3 The evolution of debt

This section focuses on the decision making process when W_t in eq. (3.2.9) is maximized again each period. Only the current young and old generation count in the decision making process. The fiscal policy preferred may depend on an agents planning horizon. Even when their preferences can be characterized by the same utility function, conflicts of interest between agents may arise if they have different planning horizons. In particular, while the elderly would prefer complete tax shifting, the young may, due to the age-independent tax structure, be interested in policies that smooth taxes over their lifetime. The tax rate set by the government follows from the maximization of eq. (3.2.9).

Assuming an interior solution, the first-order condition reads:

$$\begin{aligned}\frac{\partial W_t}{\partial \tau_t} &= \frac{\partial U_{t-1}}{\partial c_t^o}[-(1+r)s_{t-1}^*]+ \\ &\lambda(1+n)\left\{-\frac{\partial U_t}{\partial c_t^y}\left[(1-s_t^*)+(1-\tau_t)\frac{\partial s_t^*}{\partial \tau_t}\right]+\right. \\ &\left.\frac{\partial U_t}{\partial c_{t+1}^o}\left[(1-\tau_{t+1}^*)(1+r)\frac{\partial s_t^*}{\partial \tau_t}-(1+r)s_t^*\frac{\partial \tau_{t+1}^*}{\partial \tau_t}\right]\right\}=0\end{aligned} \tag{3.3.1}$$

where τ_{t+1}^* is the tax rate set by the next period government. Note that τ_{t+1}^* is a function of τ_t reflecting the fact that an incumbent government acts as a Stackelberg leader towards future governments[4]. The political preferences of the old and the young generation then follow from eq. (3.3.1) by setting λ equal to 0 or $\lambda \to \infty$, respectively. From $\lambda = 0$, *i.e.* no political weight for the current young generation, assuming positive savings and $\frac{\partial U_{t-1}}{\partial c_t^o} > 0$ it follows that $\frac{\partial W_t}{\partial \tau_t} < 0$, implying that there is no internal solution. This clearly indicates the preference for low taxes by the present elderly. Since they will have passed away the next period and do not bother about the utility of the present young generation or any other future generation, they will prefer tax rates to be set at the lowest possible level which is determined by the maximum debt level, b^{max}. The political preferences of the young follow from $\lambda \to \infty$ in eq. (3.3.1). In this case substituting eq. (3.2.3) in eq. (3.3.1) and rewriting gives:

$$-\frac{\partial \tau_{t+1}^*}{\partial \tau_t} = \frac{(1-\tau_{t+1}^*)(1-s_t^*)}{(1-\tau_t)s_t^*} \tag{3.3.2}$$

The left-hand-side of eq. (3.3.2) is associated with the marginal costs of a one-dollar tax decrease. The right-hand-side gives the willingness to substitute present taxes for future taxes. According to eq. (3.3.2), the tax policy preferred by the young is the one where marginal cost equals marginal willingness[5]. Note that since neither negative

[4]Since the next government is assumed to be a Stackelberg leader to its successors, τ_{t+1}^* implicitly takes $\tau_{t+2}^*, \ldots, \tau_\infty^*$ into account.

[5]An alternative interpretation of eq. (3.3.2) is as follows: A young individual has two ways of influencing his second-period consumption possibilities; either directly by changing his savings s or indirectly by choosing a higher (lower) current tax rate in return for a lower (higher) tax rate in the next period. The latter can be seen as 'saving through the government'. Now, eq. (3.3.2) can be interpreted as an arbitrage condition which denotes when the young consumer is indifferent between these two forms of saving. This can be seen by rewriting eq. (3.3.2) as: $\frac{-\frac{\partial \tau_{t+1}^*}{\partial \tau_t}(1+r)s_t^*}{1-s_t^*} = \frac{1-\tau_{t+1}^*}{1-\tau_t}(1+r)$ where the right-hand-side is the rate of return of private savings and the left-hand-side denotes the rate

savings rates nor tax rates or savings rates larger than 1 are allowed, for any t it will hold that $\frac{\partial\tau_{t+1}^*}{\partial\tau_t} \leq 0$. The consequence of this is that if the political power of the young is non-negligible, the government trades off current and future tax rates, given the Stackelberg assumption that it takes account of the relation between the current and all future tax rates. A lower current tax rate will lead to a higher debt inherited by the next generation which, as a result, might have to opt for a higher tax rate[6]. For old individuals the trade-off between current and future taxes is of no relevance. As noted before, if the elderly are solely decisive for the tax rate, then the tax rate will be set at its lowest possible value given the viability of the implied policy. The tax rate actually chosen in the political process will result from weighing the preferences of the old and the young generations.

Assume the utility function to be of a logarithmic type and separable, *i.e.*, $U_t = ln(c_t^y) + \theta ln(c_{t+1}^o)$, where θ is the private discount factor. Furthermore, it is assumed that government expenditures g are constant and smaller than 1[7] and that τ^{max} equals 1. Given these assumptions, it immediately follows from eq. (3.2.3) that the young choose $s_t^* = \frac{\theta}{1+\theta}$ for all t. Inserting all this in eq. (3.3.1) results in the following first-order condition for the government at time t:

$$-\theta\frac{1-\tau_{t+1}^*}{\theta(1-\tau_t)} - \lambda(1+n)\left\{\frac{1-\tau_{t+1}^*}{\theta(1-\tau_t)} + \frac{\partial\tau_{t+1}^*}{\partial\tau_t}\right\} = 0, \quad \text{for } \lambda > 0 \tag{3.3.3}$$

for $t = 1, \ldots, \infty$. Notice in eq. (3.3.3) that, as expected, the relation between the current and future tax rate is of importance only if a young individual has some political power, *i.e.*, $\lambda > 0$. The term between brackets reflects eq. (3.3.2), the marginal trade-off made by the young. However, the government also takes the preferences of the elderly, which are reflected in the first term, into account. Hence, it follows that the tax rate chosen by the government will be lower than (or at most equal to) the tax rate preferred by the young. In the extreme case where $\lambda = 0$, eq. (3.3.3) cannot hold with equality implying that there is no internal solution. As noted before, given b^{max}, the lowest possible tax

of return of 'saving through the government'.

[6]This is, however, not necessary. The tax rise may be postponed until period $t+2$ or later, leaving τ_{t+1} unaffected. In that case, the interests of the young and elderly coincide.

[7]$g < 1$ implies that the level of government expenditures per consumer is lower than the initial endowment each consumer receives in the economy.

rate results[8].

At any time t, the government, as a Stackelberg leader towards future governments knows that future governments use eq. (3.3.3) to solve for the tax rate. Therefore, it uses this equation to calculate the relation between its own and future government decisions.

To derive a solution for the infinite horizon problem the following procedure is used: first the finite horizon problem is solved and then a solution for the infinite horizon problem is derived as the limit of the finite horizon problem[9]. Assume the time horizon to be finite, $\mathcal{T}$. Obedience of the No-Ponzi-Game condition for a finite $\mathcal{T}$ implies that all public debt has to be repaid, *i.e.*[10]:

$$b_{\mathcal{T}} = 0 \tag{3.3.4}$$

The calculation executed by the forward-looking government at time t goes backward starting in the final period $\mathcal{T}$. In that period, the government has no choice but to obey the terminal condition $b_{\mathcal{T}} = 0$. The tax rate in the final period, then, follows immediately from the government's budget constraint, eq. (3.2.4):

$$\tau_{\mathcal{T}}^{*} = \frac{1+\theta}{1+(1+\hat{r})\theta}[(1+\hat{r})b_{\mathcal{T}-1} + g] \tag{3.3.5}$$

where $\hat{r} = \frac{r-n}{1+n}$, $\hat{r}$ denotes the effective interest rate, and, hence, $1+\hat{r} = \frac{1+r}{1+n}$. In period $\mathcal{T}-1$, the incumbent government explicitly takes account of eq. (3.3.5). Inserting it in its own budget constraint and taking the derivative with respect to $\tau_{\mathcal{T}-1}$ gives

$$\frac{\partial \tau_{\mathcal{T}}^{*}}{\partial \tau_{\mathcal{T}-1}} = -(1+\hat{r}) \tag{3.3.6}$$

[8]Given $b_{t-1} = 0$, this tax rate can easily be calculated to be $\tau_t = \frac{-b^{max}+g}{\frac{1+\frac{1+r}{1+n}\theta}{1+\theta}}$.

[9]Note that standard dynamic programming techniques are not applicable in this case because of the form of the target function.

[10]Strictly speaking, the No-Ponzi-Game condition for a finite horizon reads $\lim_{t\to\mathcal{T}} \left(\frac{1+r}{1+n}\right)^{t} b_t = 0$. But since $\left(\frac{1+r}{1+n}\right)^t > 0$ for every finite t this implies $b_{\mathcal{T}} = 0$.

Inserting eqs. (3.3.5) and (3.3.6) into eq. (3.3.3) and solving for τ_{T-1} learns that an interior solution will be obtained if debt $b_{T-2} \leq b_T^{max} = -\frac{1+(1+\hat{r})}{(1+\hat{r})^2} g + \frac{[1+(1+\hat{r})\theta][(1+\hat{r})+1]}{(1+\theta)(1+\hat{r})^2}$. If $b_{T-2} = b_T^{max}$ the tax rate will be equal to τ^{max}. Moreover, $b_{T-2} > b_T^{max}$ is simply not possible. The (foreign) lenders will prevent this level of debt from being issued since it can never be repaid. Therefore, an interior solution is always obtained. Given τ_{T-1} thus obtained, the solution for the tax rate τ_{T-2}, τ_{T-3} and so on can be obtained in the same way.

The solution for the infinite-horizon analogue of the model follows by taking the limit of the T period model[11]. The solution for the tax rate at time t for the infinite-horizon model then reads:

$$\tau_t^* = \begin{cases} \tau_t^{int} & if \quad b_{t-1} \leq b^{max} \\ \\ \tau^{max} & if \quad b_{t-1} = b^{max} \end{cases} \tag{3.3.7}$$

where

$$\tau_t^{int} = 1 - \frac{(1+\theta)(1+\hat{r})}{1+(1+\hat{r})\theta} \cdot \frac{\beta}{1+\beta}[b^{max} - b_{t-1}] \tag{3.3.8}$$

and

$$b^{max} = -\frac{g}{\hat{r}} + \frac{1+(1+\hat{r})\theta}{(1+\theta)\hat{r}} \tag{3.3.9}$$

where τ_t^{int} is the interior solution for the tax rate at time t. β is defined as $\frac{\lambda(1+n)+\theta}{\lambda(1+n)\theta}$ and describes the political compromise between the preferences of the present generations. It reflects the political weight in the decision making process attached to present utility (of the old and young generation) relative to future utility (of the present young). Of course, b^{max} is independent of the political influence of both generations as measured by λ. Rewriting b^{max} as $\frac{\theta}{1+\theta} + \frac{1-g}{\hat{r}}$, it becomes clear that, given the assumption $g < 1$, public debt can exceed savings which, in that case, implies that the country is a debtor

[11]That the limit of the solution of the finite-horizon problem is indeed a solution for the infinite-horizon problem is easily seen by writing down the first-order conditions of the infinite-horizon model and checking the candidate solution and, furthermore, noting that the finite-horizon solution behaves as a turnpike (see *e.g.* Blanchard and Fischer, 1989), hence there are no transversality problems.

in the world capital market[12].

The evolution of debt can be traced by substituting solution (3.3.7) into the budget restriction (3.2.4). This gives the following linear difference equation:

$$(b^{max} - b_t) = \frac{1+\hat{r}}{1+\beta}(b^{max} - b_{t-1}) \tag{3.3.10}$$

$(b^{max} - b_{t-1})$ can be interpreted as the scope politicians in period t have for increasing the inherited debt. According to eq. (3.3.10) this scope for policy-making changes from one period to another by the factor $\frac{1+\hat{r}}{1+\beta}$. From this, it can be concluded that debt (and thus the tax rate) increases if $\hat{r} < \beta$, decreases if $\hat{r} > \beta$[13] or remains unchanged if $\hat{r} = \beta$. An intuition for this result can be given by noticing that eq. (3.3.3) implies equality between the marginal political willingness to substitute taxes and the marginal cost of tax substitution. The willingness in the political process to substitute present for future taxes is given by the first two terms in eq. (3.3.3):

$$\mathrm{MRS}_{\tau_t^* \tau_{t+1}^*} = \frac{\partial W_t}{\partial \tau_t^*} \Big/ \frac{\partial W_t}{\partial \tau_{t+1}^*} = \frac{1-\tau_{t+1}^*}{1-\tau_t^*}\beta \tag{3.3.11}$$

From the definition of β it can be seen immediately that this willingness depends on both the private discount factor θ and the political power balance $\lambda(1+n)$. Lower values for θ imply more impatience on the side of the individual (young) consumers. They are more interested in present than in future consumption. This, obviously, lowers the political willingness to give up present consumption (and thus present utility) through higher

[12] If, instead of an open economy, a closed economy with a linear production technology is assumed (*i.e.* $f(k) = rk + w$ where w is normalized: $w \equiv 1$) the capital market imposes an additional constraint $b \leq s = \frac{\theta}{1+\theta}$. In that case, $b^{max} = min\{\frac{\theta}{1+\theta} + \frac{1-g}{\hat{r}}, \frac{\theta}{1+\theta}\}$, which implies that the constraint imposed by the tax base is redundant if $g < 1$. This implies a lower upper limit for public debt than in the open economy case, thus altering the set of feasible policies. However, in case of $g > 1$ (but smaller than $1 + \hat{r}\frac{\theta}{1+\theta}$, national income), the results for the closed economy coincide with those of the small open economy.

[13] Here, an unstable process occurs where the debt and the tax rate tend to minus infinity. In principle, negative debt also poses a problem of viability: in the case of foreign borrowers, the rest of the world must be willing to serve it. It is possible to avoid such a process by imposing *e.g.* a non-negativity restriction on the tax rate. Then the debt will converge to a finite minimum level. Calculations for this case have appeared in an earlier version of this chapter and are available upon request. Since they do not provide additional insights they are left out here for expositional reasons.

taxes now in exchange for future consumption. A similar argument holds for a lower $\lambda(1+n)$, *i.e.* more political weight for the present elderly. Then again, the willingness to give up present consumption in exchange for future consumption is lower. On the other hand, the marginal cost of tax shifting is given by

$$\mathrm{MRT}_{\tau_t^* \tau_{t+1}^*} = -\frac{\partial \tau_{t+1}^*}{\partial \tau_t^*} = (1+\hat{r})\frac{\beta}{1+\beta} \tag{3.3.12}$$

Note that this marginal cost is lower than the marginal cost of a debt increase when debt has to be redeemed completely in the next period. In that case, the marginal cost of a one-dollar debt increase (or, equivalently, a one-dollar tax decrease) would be $1+\hat{r}$ (see eq. (3.3.6)). The reason for this is that, when debt is not completely redeemed in the next period, it is possible to shift the burden of the one-dollar debt increase partly over to future generations. This decreases the marginal costs. The degree to which this possibility is used depends on the private weight attached to future consumption (θ) and the political power balance ($\lambda(1+n)$) as can be seen from β. In the optimum the marginal costs of tax shifting have to equal the marginal benefits. From equalizing these two it follows that the tax rate (and, hence, debt) increases if $\hat{r} < \beta$, decreases if $\hat{r} > \beta$ or remains unchanged if $\hat{r} = \beta$.

The following proposition summarizes the results:

Proposition 3.1 *If $b_0 < b^{max}$, $\lambda > 0$ and $\theta > 0$, for every finite t, it holds that:*

(a) If $\hat{r} < \beta$ then

$$b_{t-1} < b_t$$
$$\tau_{t-1}^* < \tau_t^*$$

(b) If $\hat{r} > \beta$ then

$$b_{t-1} > b_t$$
$$\tau_{t-1}^* > \tau_t^*$$

(c) If $\hat{r} = \beta$ then

$$b_{t-1} = b_t$$
$$\tau_{t-1}^* = \tau_t^*$$

Insert Figure 3.1

Figure 3.1 shows the evolution of the debt (eq. (3.3.10)). Case (a) is the case $\hat{r} < \beta$ and the system converges towards the steady state b^{max} through a sequence of increasing levels of debt. Case (b) for which $\hat{r} > \beta$ results in an ever decreasing debt. The 45°-line corresponds with regime (c) in the proposition where $b_t = b_0 \equiv b$ for all t. The adjoining tax rate is then given by $\tau = \frac{1+\theta}{1+(1+\hat{r})\theta}[\hat{r}b + g]$.

It is interesting to compare the evolution of the debt described in Proposition 1 with the evolution that would result from the maximization of a social welfare function. It can easily be shown[14] that, if the government maximizes a Benthamite social welfare function with a social discount factor ρ, a proposition analogous to Proposition 1 with β replaced by $\frac{1}{\rho} - 1$ can be derived. $\frac{1}{\rho} - 1$ is the social rate of time preference. So, any evolution of debt and taxes that can result from decision making by a representative government can also be found by maximizing a social welfare function that explicitly takes account of the utility of future generations. So, a representative democracy, where decision making is based on the utility of the currently living generations only, may act as if it takes the welfare of all future generations into account. However, it will do this only to a limited extent: since $\beta > 1$ the implicit social rate of time preference can never be below 1. This result provides a justification for the common practice to mimic a positive government by a normative social welfare maximizing government with a high social rate of time preference compared to the individual rate of time preference (see *e.g.* Van der Ploeg and Van de Klundert (1991)).

3.4 The effects of parameter changes

The analysis in the previous section led to a description of the evolution of the debt as a function of the exogenous parameters λ, r, θ, g and n. This section analyses how this evolution is affected by a change in these parameters[15]. Two cases have to be dis-

[14] See Meijdam, van de Ven and Verbon (1994).

[15] Notice the difference with a comparative statics analysis where changes in the stationary state are analysed. Here, changes in the whole time path of debt due to changes in the exogenous parameters are

tinguished: one where the change in the parameters is unanticipated, the other where it is anticipated. In case of an unanticipated change in the parameters these effects remain unknown until the time of the change, $t = \tilde{t}$. If, however, the change is anticipated, the effects are taken into account from period 1 onwards as the news of the change in the parameters arrives[16]. From eq. (3.3.10) it follows that, given b_{t-1}, a change in λ, n, θ, r or g in period t has two effects: one running through b^{max}, changing the scope for policy making, the other running through $\frac{1+\hat{r}}{1+\beta}$, affecting the factor at which the scope for policy making is changing from one period to another. The following table summarizes the effects on b^{max} and $\frac{1+\hat{r}}{1+\beta}$:

	$\partial\lambda$	$\partial\theta$	∂r	∂g	∂n
∂b^{max}	0	+	$\pm$	$-$	+
$\partial\frac{1+\hat{r}}{1+\beta}$	+	+	+	0	$-$

In the sequel changes in the exogenous variables at some time $\tilde{t}$ will be investigated. The reference situation is $\hat{r} < \beta$, *i.e.* increasing debt levels and tax rates.

Effect of an increase in political power of an old individual. This is reflected by a decrease in λ at time $\tilde{t}$. Since b^{max} is independent of political elements, this does not affect the maximum level of debt. However, this maximum sustainable level is approached faster. Because the main concern of the elderly is a low tax rate, each period a larger part of the scope for policy-making is consumed. The implications of this are represented in Figure 3.2[17]. In the case of an unanticipated decrease in λ, debt jumps to a higher level at the time of the change, $\tilde{t}$. Due to this jump, the old generation present at that time enjoys large gains from its increased political power. Subsequent generations

analysed.

[16]Note that, if the debt level at the time the news of the change arrives (either at $t = 1$ or at $t = \tilde{t}$, the time of the change) is larger than the new b^{max} a situation occurs where debt can never be repaid and the government is in fact bankrupt.

[17]In this figure and the figures to come, the dotted lines refer to the situation where there is no change in the respective exogenous parameter. The dashed line denotes the debt path followed in this case. The shift of the axis is due to a change in b^{max} changing the scope for policy-making. Illustrated is the case where $\tilde{t} = 2$. Hence, in the initial period 0, $b^{max} - b_0$ is given by point **a**. If there was no change, the next period, $b^{max} - b_1$ would be given by point **b**. However, due to the change in λ, it is given by point **b'**. Etcetera.

Insert Figure 3.2

of elderly can profit from their increased political power to a much lower degree, because they are left with a debt level much closer to the maximum level b^{max}. Moreover, each generation reinforces this effect for their successors by consuming a larger part of their scope for policy-making.

In case of an anticipated change in the political power of the elderly, the generations at $t = 1$ will immediately exploit the possibility to jump to a higher debt level which arises due to the change in political power at $\tilde{t}$[18]. Note that, because of this jump to a higher debt level and the chosen debt policy, all generations of elderly with increased political power, including the elderly of generation $\tilde{t}$, are confronted with a smaller scope for policy-making. Hence, the gains from their increased political power are relatively low.

Effect of an increase in the private discount factor. If the private discount factor θ increases, two opposing forces are at work as can be seen from the table above. First, due to higher savings, the tax base enlarges making lower taxes and larger debt levels possible (*i.e.* b^{max} increases, widening, ceteris paribus, the scope for policy-making). Second, the relative political weight of present utility decreases (*i.e.* β decreases) implying, ceteris paribus, a preference for higher taxes now relative to future taxes. As a result, less debt is passed on to the future thus increasing the scope for policy-making for future generations of politicians. Figure 3.3 provides an illustration of an unanticipated

Insert Figure 3.3

and an anticipated change. In case of an unanticipated increase in θ, the two opposing effects are at work at the same time $\tilde{t}$. Therefore, the total effect is ambiguous and de-

[18]The generations between $t = 1$ and $t = \tilde{t}$ then pursue debt policies that would be unsustainable if they were continued after $\tilde{t}$. This can easily be seen from Figure 3.2 by extending the solid line of policies between 0 and $\tilde{t} - 1$. It will not pass through the origin. However, because of the change in political power at $\tilde{t}$, a switch is made towards a sustainable policy.

pends on the magnitude of the increase[19]. For small alterations in θ the tax-base effect dominates, for larger alterations in θ the political weight effect dominates which even might lead to a regime switch from increasing debt levels to decreasing debt levels. The upper part of Figure 3.3 illustrates a jump to a lower level $b_{\tilde{t}}$. Savings increase, making more debt creation possible. However, politicians are not inclined to do so, due to the higher political weight attached to future utility. If there is an anticipated change, the generations before time $\tilde{t}$ can and will take advantage of the enlargement of b^{max} due to the increase in θ at time $\tilde{t}$. Hence, at $t = 1$ there is jump to a higher level of debt. At time $\tilde{t}$, only the increased political weight of the future is left, implying a slower growth of the debt. Note that the generations at $t = 1$ only experience the enlargement of the tax base and not the change in political weights. Therefore, they partly exploit the increase in the tax base. Due to this anticipation effect, the generations at $\tilde{t}$ profit only partly from the widening of the tax base.

Effect of an increase in the interest rate. An increase in the interest rate has two effects. First, it enlarges the interest burden of existing debt and makes debt creation in the future more costly. Second, the interest revenues on savings rise which leads to a widening of the tax base. Clearly, these two effects are opposing. The increase in the interest burden necessitates, ceteris paribus, lower debt growth. The effect on the tax base makes, (again) ceteris paribus, larger debt levels possible. These effects can be derived from eq. (3.3.10). The increased interest burden of existing debt necessitates an increase of the factor, $\frac{1+\hat{r}}{1+\beta}$, by which the scope for policy-making changes. The higher costs of future debt issuance and the growth of the tax base are both contained in the effect on b^{max}. From rewriting b^{max} as $\frac{\theta}{1+\theta} + \frac{1-g}{\hat{r}}$ it immediately follows that $\frac{\partial b^{max}}{\partial r} < 0$ since $g < 1$. So, the positive tax-base effect of an increase in r is dominated by the negative interest-burden effect. An increase in r leads to a jump to a lower level of debt,

Insert Figure 3.4

either at $t = \tilde{t}$ (unanticipated, the upper part of Figure 3.4) or at $t = 1$ (anticipated, the

[19]Note that, if $\hat{r} > \beta$, the political weight effect always dominates and a jump at time $\tilde{t}$ to a lower level of public debt can be observed.

lower part of Figure 3.4)[20]. After $\tilde{t}$ the erosion of the scope for policy-making is slowed down because of the increased interest burden of inherited debt, implying less debt to be passed on to future generations.

Effect of an increase in government expenditures. The effect of a change in the government expenditures on b^{max} is clearly negative. An increase in g means that a larger share of tax revenues has to be used for financing these expenditures, leaving less room for debt repayments. This naturally implies that less debt can be accumulated, *i.e.* b^{max} is lower. The growth rate of debt is unaffected since the political weights and the interest rate remain unchanged. Hence, an increase in g leads to a negative jump in debt. When this jump is observed depends on whether it is anticipated or unanticipated. An anticipated change in g at time $\tilde{t}$ leads to a jump at $t = 1$, an unanticipated change leads to a jump at $t = \tilde{t}$. (Figure 3.5).

Insert Figure 3.5

Effect of a decrease in population growth. The effect of a decrease in population growth, *i.e.* $\partial n < 0$, comprises two effects: an increase in the political power of the old generation ($\partial \lambda(1+n) < 0$), because of the increase of their relative number, and an increase in the effective interest rate ($\partial \hat{r} > 0$). The world interest rate r, of course, remains fixed, just as the political influence of a young individual, λ. Rewriting eq. (3.3.10) gives $b^{max} - b_t = \frac{1+\hat{r}}{\frac{1+\theta}{\theta}(1+n)+\frac{1}{\lambda}}(b^{max} - b_{t-1})$. Therefore, a decrease in n implies that the scope for policy-making is diminished less quickly from one period to another by successive governments. Apparently, the effect of an increase in the political power of the elderly is dominated by the effect of the increase in the effective interest rate. Hence, ceteris paribus, a smaller part of the scope for policy-making is used by the present generation of politicians. Assuming $g < 1$, this effect is reinforced because a

[20] A jump to a lower debt level at $t = 1$ in anticipation of a future increase of the interest rate may seem odd, since consumers do not exhibit altruistic behaviour towards future generations. So why should they bother? The reason for this is, of course, that, since they are Stackelberg leaders towards future generations, they know the behaviour of generation $\tilde{t}$ in response to the change in the interest rate. This causes a reaction of generation $\tilde{t}-1$ to the displayed behaviour of generation $\tilde{t}$ which in turn causes a reaction of generation $\tilde{t}-2$, etcetera.

decrease in n leads, through the increase in the effective interest rate, to a decrease in b^{max}, implying a contraction of the scope for policy-making. Lower population growth lowers the tax base and by this b^{max} decreases. Figure 3.6 gives an illustration of an

Insert Figure 3.6

unanticipated (upper part) and an anticipated (lower part) decrease in the population growth rate. The decrease in b^{max} necessitates a jump to a lower level of debt, either at period 1 (anticipated) or at period $\tilde{t}$ (unanticipated).

Parameter changes and actual debt policy

Returning to the actual evolution of debt after World War II described in the Introduction, it is interesting to check whether this evolution can be explained by the model through parameter changes. However, before continuing, some qualifications should be made. In this model, there is no growth while, in reality, economics have grown. Also, with all parameters constant, b either converges to b^{max} or declines without bounds. Hence, the following analysis should be read keeping this in mind.

With respect to the initially declining debt levels, according to Proposition 1, this can be the case if β is lower than $\hat{r}$. Although the effective interest rate was quite low in that period, the discount factor may have been so high that β was even lower. Indications of this may be found in the fact that savings by consumers were relatively high given the low interest rate. Moreover, in this same period many funded pension plans were initiated, also indicating the relatively high importance the generations of that time attached to the future. Starting in the sixties, the post-war baby-boom generation started to enter the labor market. In terms of this model this implies a (small) increase in n. It might be hypothesized that this generation was more consumption minded than the previous one witnessed by the fact that savings did not increase in spite of the increase in the interest rate starting in the early seventies. So, a larger generation with a lower discount factor entered the labor market. According to the model, this might have caused the shift from decreasing to increasing debt levels as observed in practice. This growth of debt continued during the eighties leading to rather high levels of debt. Recently, in several countries policy measures were enacted to mitigate the growth rate of debt. Moreover, as expressed by the EMU-norm regarding the required debt ratio for the member states of the EC, maximum levels for the debt ratio were fixed. These phenomena can be interpreted in the framework of the model. In particular, many EC-countries expect an ageing of their population. As noted before, such a decrease in population growth should lead to a lower maximum level of debt. Moreover, this maximum level should be approached at a slower pace. Figure 3.7 illustrates this effect of an anticipated decrease in population growth[21]. In this figure, the generation of politicians at time $t = 2$ expect

Insert Figure 3.7

an ageing of the population to occur at time $t = 3$. Moreover, the generation that is young at $t = 2$ is relatively large. In the simulation $n = 0.3$ was taken in the first two periods to indicate an increasing population, followed by a period where the dependency ratio increases ($n = -0.1$ for $t = 3$) after which the population stabilizes ($n = 0$). If the length of one period were equal to 25 years, and period 1 would correspond with the first 25 years after World War II, this stylized demographic development would roughly correspond with the current EC-situation. The population is expected to stabilize at a lower level than the current one in the second or third decade of the next century, after a once-only increase of the dependency ratio due to the retirement of the post-war baby boom[22]. It appears that, in anticipation of the decrease in population growth, debt jumps to a lower level at time $t = 2$. After this, debt gradually approaches the new maximum level. Notice from Figure 3.7 that if the population would stabilize after $t = 2$ instead after $t = 3$, the debt ratio would be slightly higher in these periods. So, the baby boom, *i.e.* the increase in the dependency ratio at $t = 3$ ($n = -0.1$), hardly has any effect on the evolution of debt. But the baby bust, *i.e.* the permanent decrease in population growth ($n = 0$) affects the evolution of debt to a large degree.

[21]In the simulation the following parameter values are used: $\lambda = 0.55$, $g = 0.5$, $\theta = 0.5$, $r = 2.1$, $b_0 = 0$.

[22]Notice from Figure 3.7 that an increasing debt ratio has been assumed for the whole post-war period. In this simulation the factors that contributed to a declining debt ratio during this period are not taken into account.

3.5 Summary and concluding remarks

In the literature surveyed in the introduction dealing with the strategic use of public debt, the driving force behind these models were the differences in preferences with respect to the size and composition of public expenditures between successive governments. In the present model, the effect of public debt on future decisions is also taken into account but for a different reason. Current young generations are still alive in the following period and future policy choices are relevant to them. Current old generations clearly prefer full tax shifting since they are no longer alive in future periods. Thus, the driving force behind the current model is the difference in time horizon of the different generations present.

An overlapping-generations model is used. Consumers, both young and old, pay a consumption tax at the same rate. Given logarithmic utility functions, explicit solutions for the development of the tax rate and the level of debt are derived. A sequence of representative governments will gradually increase or decrease debt. In the former case, debt converges to a level determined by the maximum taxing capacity. In the case of decreasing debt, it may converge to a level determined by some lower bound on the tax rate. Under a social planner, debt *may* take the same route as under a representative government. However, even though differences in preferences are abstracted, this is by no means necessary. Representative governments may choose another time path of debt than a social planner because of the inherent tendency of consumers to shift taxes to future generations.

In this model the time path of debt can be calculated completely. Therefore, it is possible to derive how this path will change due to some foreseen or unforeseen change in the parameters. Such a change leads to a jump in the level of debt and, thus, forcing it to follow another path. This new path may converge towards a different maximum level, or may even decrease instead of increase. The following comparative-dynamics results are worth pointing out here. First, if savings increase due to a lower rate of time preference, this can be concomitant with a desired decrease in the level of public debt. An increase in savings, giving a higher potential to finance public debt, is at the same time an expression of the increased weight that the current young generation places on its future consumption. By the same reasoning, an increase in the rate of time preference leads to an increase in the level of public debt. Second, in this model anticipated ageing of the population leads to an immediate curtailing of debt. Basically, ageing limits the possibility to shift the tax burden to the future by debt creation as it increases the effective interest rate.

Some qualifications are in place. First, when using the model to analyse actual debt policy, one should keep in mind that in the present model growth is absent. In reality, of course, economies did grow, at different points in time at different rates, after World War II. Assuming constant growth over time would not change the results dramatically. It would increase the tax base, and thus b^{max}. Besides, the current young would have a higher initial endowment to start with. Given the unchanged political weights, the conjecture is that in this case the government would redistribute from the current young to the current elderly and, therefore, debt would increase faster. Second, debt either tends to its maximum level, determined by the maximum tax capacity, or to a minimum level (not explicitly considered here). Hence, there is no internal solution determined by the political weights.

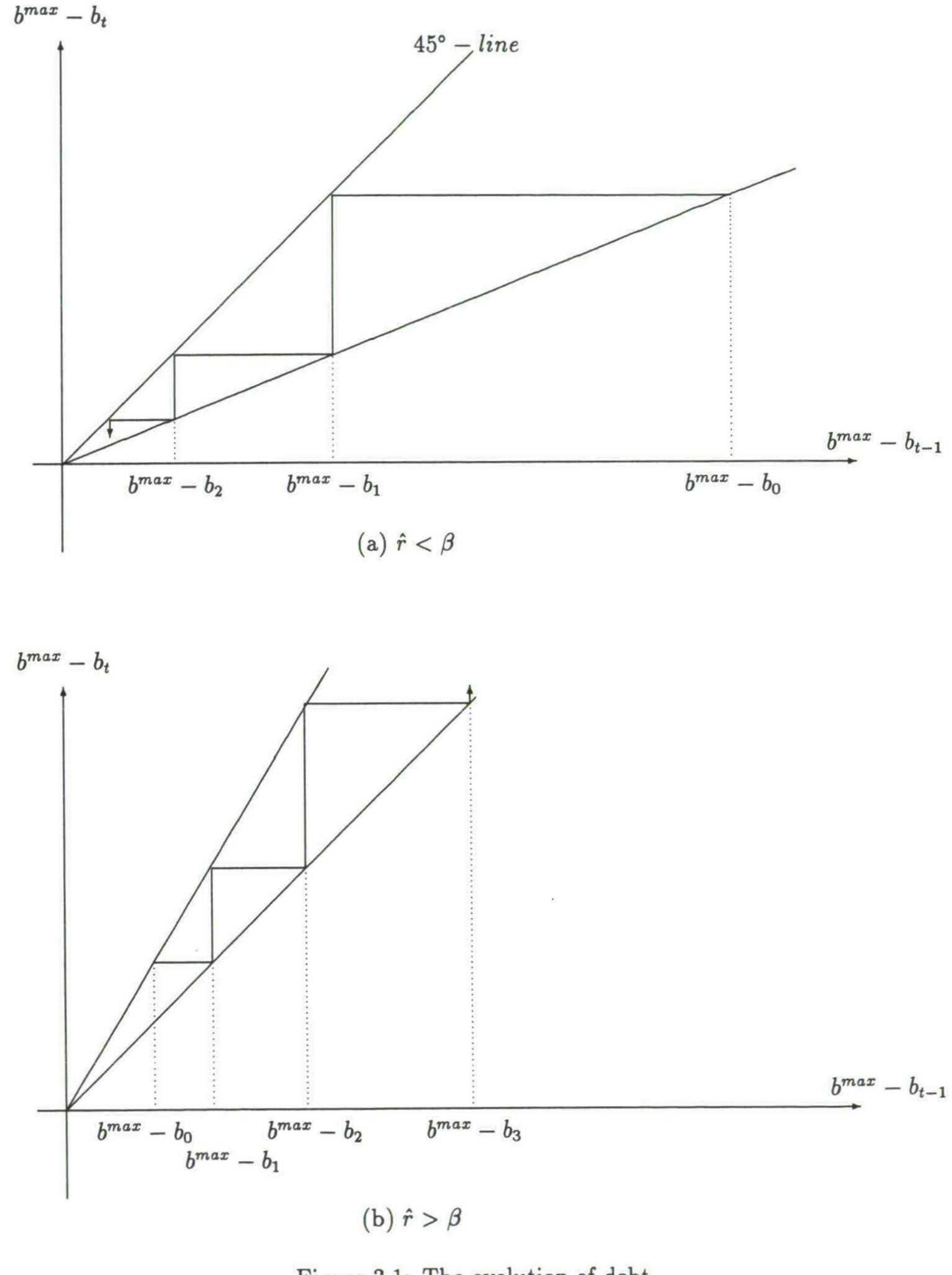
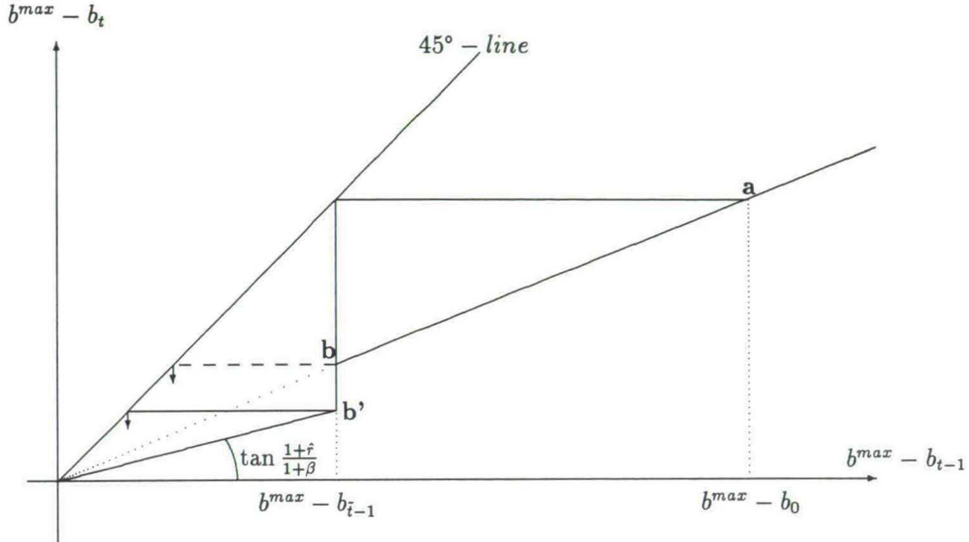
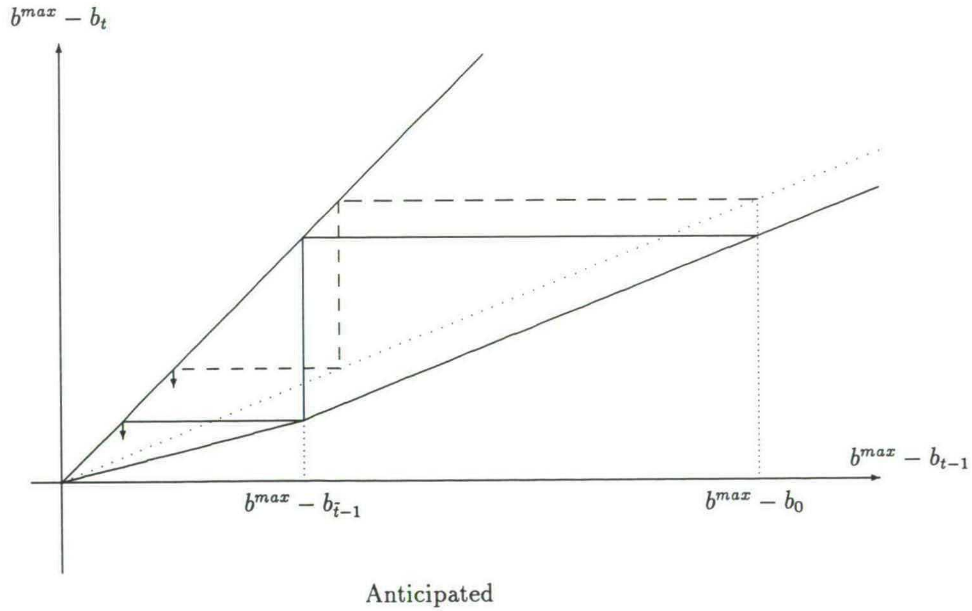


Figure 3.1: The evolution of debt



Unanticipated



Anticipated

Figure 3.2: $\partial\lambda < 0$

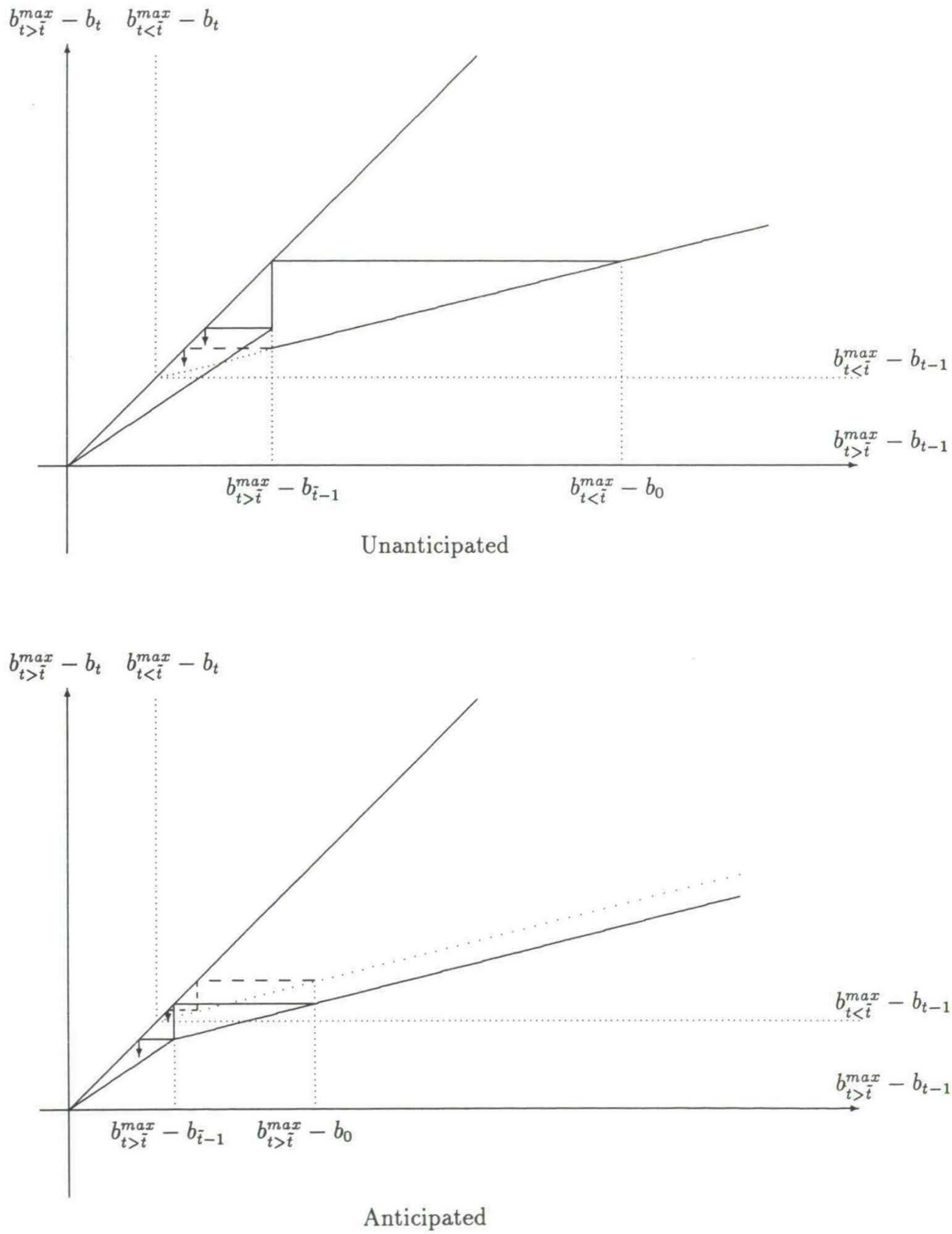
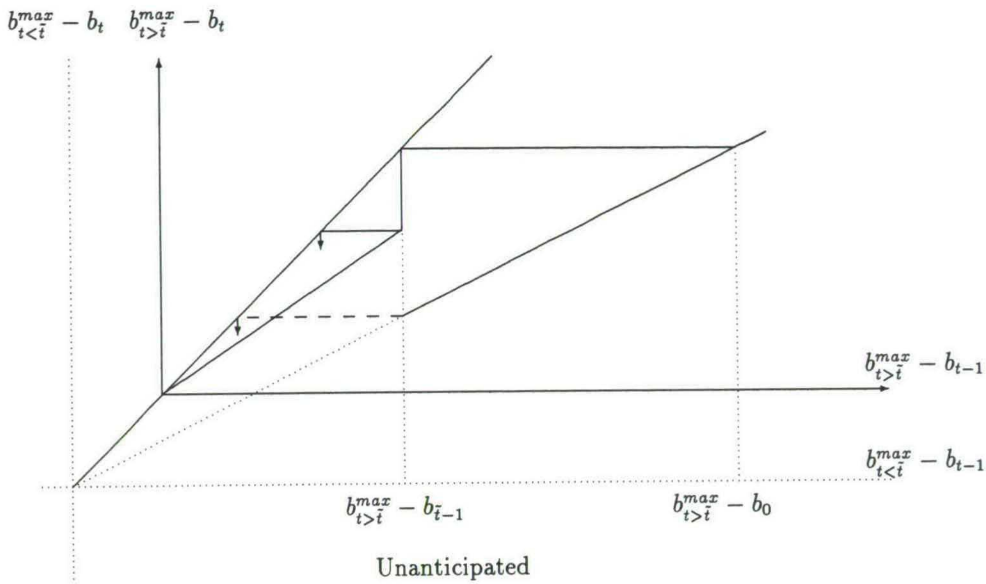
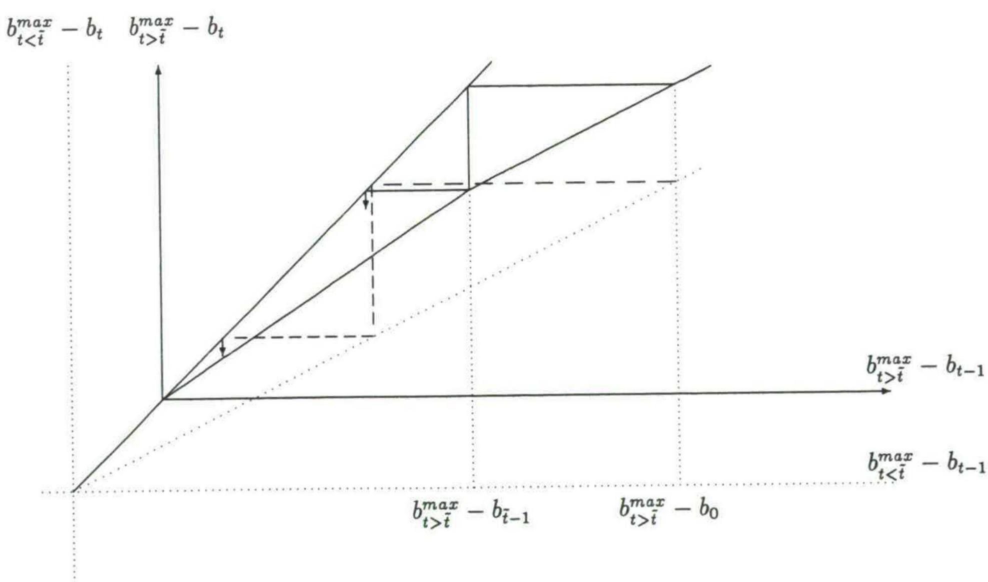


Figure 3.3: $\partial\theta > 0$

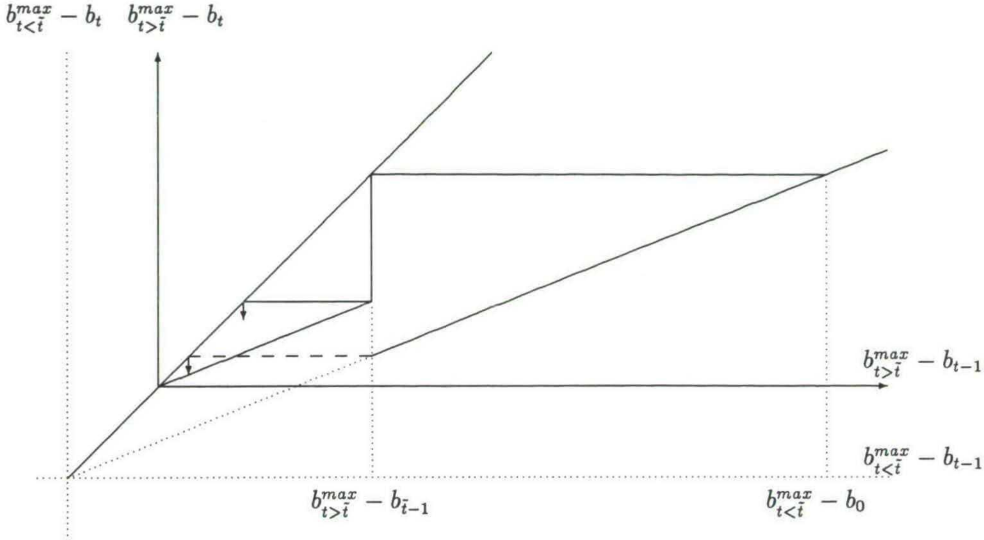


Unanticipated

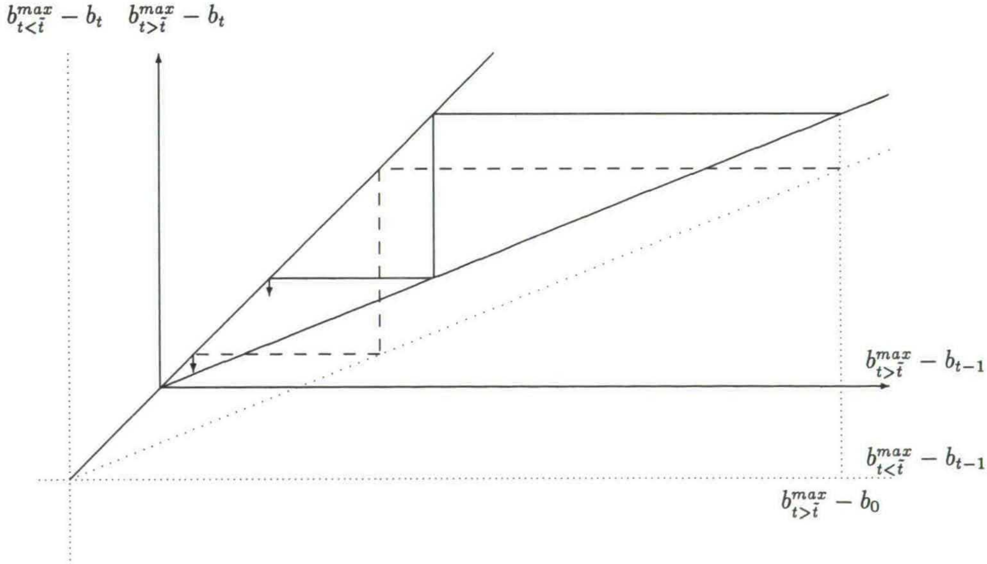


Anticipated

Figure 3.4: $\partial r > 0$

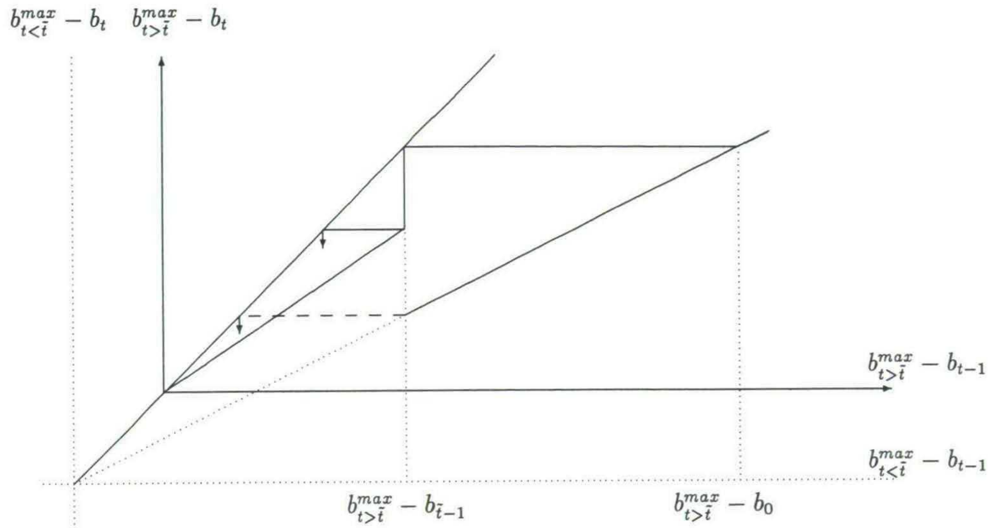


Unanticipated

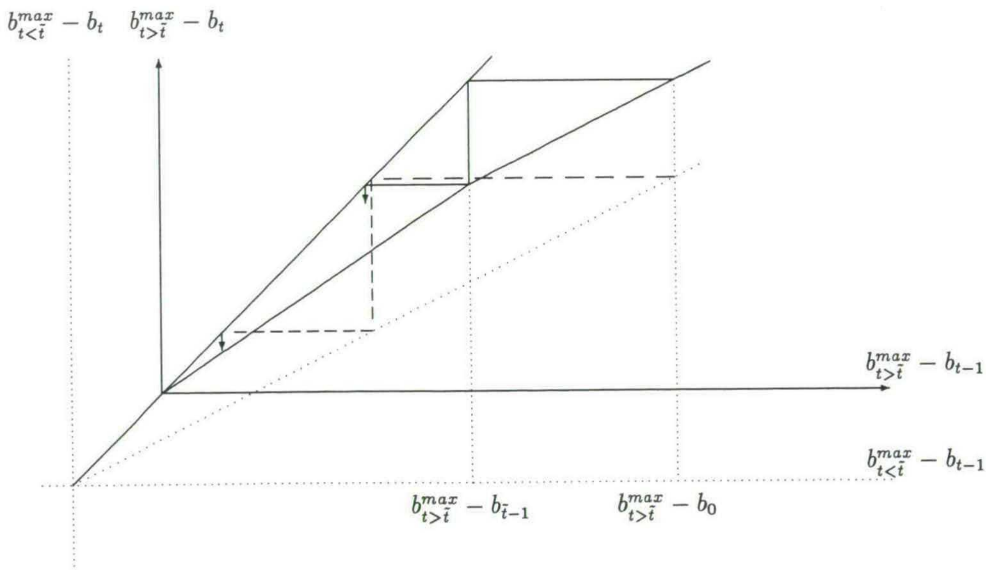


Anticipated

Figure 3.5: $\partial g > 0$



Unanticipated



Anticipated

Figure 3.6: $\partial n < 0$

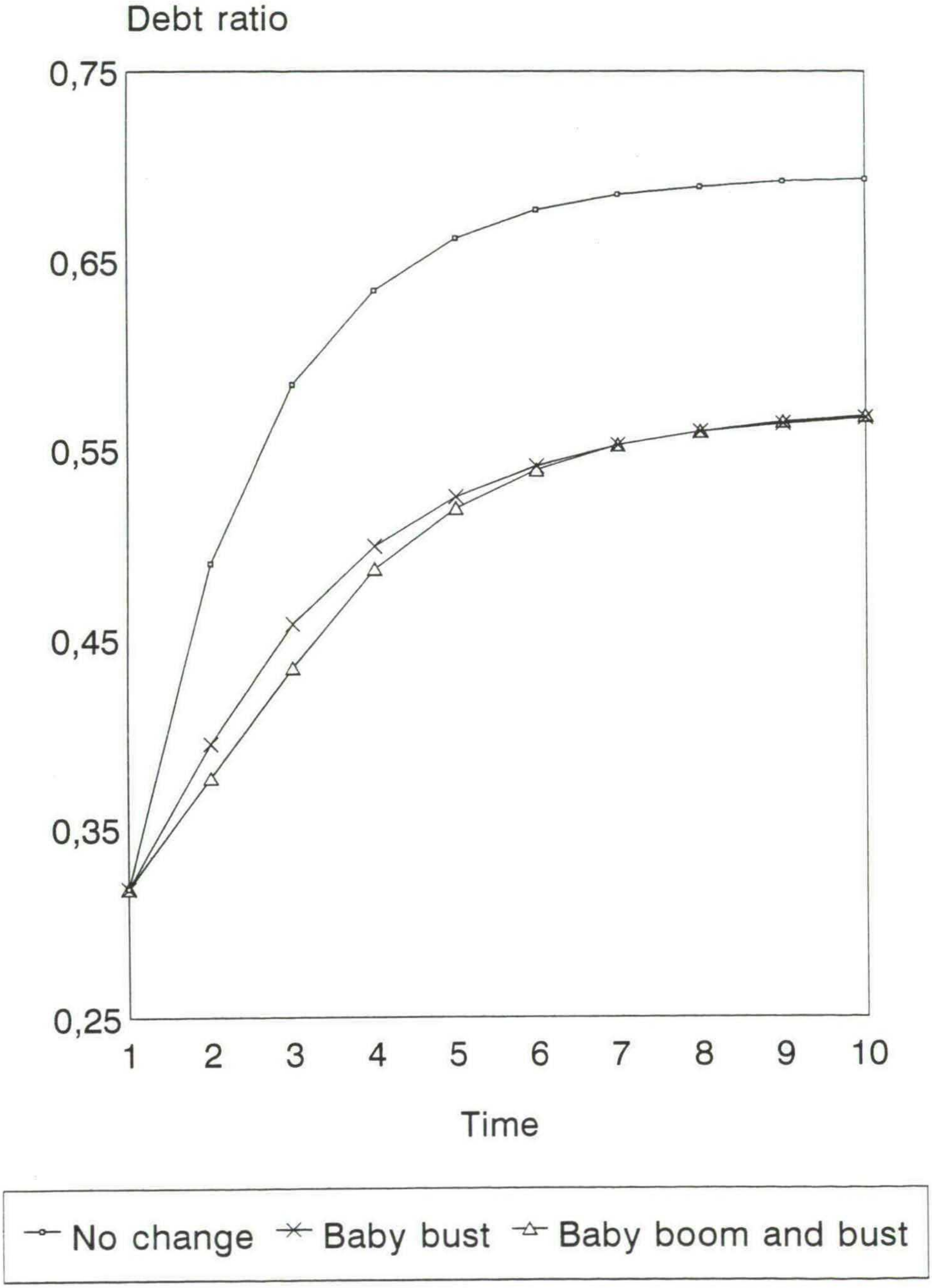


Figure 3.7: The evolution of debt

Chapter 4

The evolution of public pensions

4.1 Introduction

In most countries public pensions are financed by a pay-as-you-go (PAYG) system. Private pensions, on the other hand, are generally financed by a capital reserve (CR) system. In a PAYG system, the pension transfers to the current old generation are financed by tax payments of the current young generation. A CR system implies that generations save for their own old age.

Several explanations are given why PAYG systems exist. Most authors model the level of taxes and transfers as the outcome of some voting process within an overlapping generations framework. Several groups of agents, having different preferences about the level of government transfers, decide on this level by some form of voting. Browning (1975) and Boadway and Wildasin (1989) use a median voter framework to analyse the determination of taxes and transfers in a PAYG system. In Browning (1975), this results in a social insurance budget which is "too large" in a steady state. The reason for this result is that the optimal level of social insurance, *i.e.* the level that maximizes lifetime utility, is the level preferred by the youngest voter. The median voter, however, prefers a higher level. However, Browning (1975) neglects the existence of private savings which might offset the overexpansion of the social insurance budget. The existence of private savings is taken into account by Boadway and Wildasin (1989). In their model, agents face borrowing constraints on the capital market. The tax-transfer scheme initially overshoots the steady-state value of the system because expectations are adaptive. Then, depending on the parameters of the system, the tax-transfer scheme gradually converges towards the steady state by a pattern of alternative over- and undershooting, or the scheme does not converge but cycles around the steady state.

Both Browning (1975) and Boadway and Wildasin (1989) abstract from the notion of altruism between generations. Altruism is an important motivation for the existence of a PAYG system in Hansson and Stuart (1989) and in Veall (1986). In Hansson and Stuart (1989), current generations choose some path of taxes and transfers to which future generations are committed by law. Each period the present generations may amend the law but they will choose not to do so. The reason is that each present old generation will be opposed to every amendment to the present social security law and will block this amendment since each present generation has veto power. Veall (1986) treats social security as a trade between present and future generations in which present generations take the decisions of future generations on social security into account. In the model used by Veall (1986), the young are solely responsible for the level of taxes and transfers. Because of their altruism towards the older generation, they are willing to make a transfer. In a paper by Verbon and Verhoeven (1992), the decision on the level of transfers is made in a representative democracy. *I.e.*, the government is modelled as a group of politicians representing the interests of the present living generations only. The level of transfers that is chosen depends on the political influence of the elderly and the young. This can also be interpreted as a case of altruism, where the young generation, as in Veall (1986), is solely responsible for the level of taxes and transfers, and they are altruistic towards the current elderly. Also a social contract may explain the existence of PAYG public pension systems. Current young generations give a transfer to the current old because, if they do not give this transfer, the next-period young generation is not obliged to give a transfer to the current young when they are retired. It then may be beneficial to comply with the social contract (Sjoblom (1985), Verhoeven (1993)).

The importance of future decisions for present tax and transfer levels is evident. Rational present young generations form an expectation about their future pension transfers since this influences their decision about how much to save now. The involvement of future decisions greatly complicates matters. To be able to deal with these complications simplifying assumptions are used. In Browning (1975) present generations decide on a tax-transfer policy based on the assumption that this policy is not changed in the future. Since future generations make the same assumption this turns out to be correct. Hansson and Stuart (1989) assume that the tax-transfer policy is laid down in the constitution and each generation has veto power over the amendments. Others use behavioural assumptions towards future decisions. In Verbon and Verhoeven (1992) present generations display Nash behaviour towards future generations, *i.e.* present generations take future decisions on taxes and transfers as given. Veall (1986) makes the (inconsistent) assumption of present generations displaying Stackelberg behaviour towards future generations, *i.e.* taking the behavioural responses of future generations on present decisions as given, while future generations display Nash behaviour towards their succeeding generations[1]. An important aspect of this chapter is that all generations, *i.e.* present and future, display Stackelberg behaviour towards their succeeding generations. Because of the sequential order in which the decisions on the pension transfers are made, this is the most natural assumption.

This chapter uses a framework of overlapping non-altruistic generations which is given in Section 4.2. The young decide on how much to save, but, because they are rational, this decision depends on the level of transfers both when they are young, since then they have to finance these transfers, and when they are old, since then they are the recipients of these transfers. The level of transfers is chosen by the government. As in Verbon and Verhoeven (1992), the government is modelled in a representative democracy setting. As noted before, and different from their assumption of Nash behaviour towards future generations, Stackelberg behaviour of present and future generations towards their succeeding generations is assumed. Additional differences are: First, Verbon and Verhoeven (1992) exclude negative savings and transfers. Their solution is therefore given by a constrained steady state with either zero savings or zero transfers. Second, their economy can be dynamically efficient as well as inefficient. But, because of the constraints, this is not really important for their model. In Section 4.3, the differences between Nash behaviour and Stackelberg behaviour are analysed. Besides, section 4.3 derives an analytical solution for the representative democracy and gives a description of the evolution. Furthermore, the welfare consequences are analysed for the initial gener-

[1]The inconsistency then arises because the generations of the next period are assumed to behave the same, *i.e.* play Stackelberg towards their successors and assume future generations to play Nash.

ations if the government decides to introduce a PAYG pension system. Also the relation between the solution under the assumption of Nash behaviour and the solution under the assumption of Stackelberg behaviour is analysed. In addition, a comparison is made between this solution and the policy chosen by a government optimizing a Benthamite social welfare function. Section 4.4 analyses changes in the exogenous parameters of the model. The chapter is concluded by Section 4.5 with a summary and some additional remarks.

4.2 The Model

This section develops an overlapping-generations model of a small open economy with a private sector and a government sector.

The private sector

The private sector consists of agents living for two periods. The population grows at a constant rate n. The first period of their lifes the agents are endowed with one unit of income of which an amount of $\frac{p_t}{1+n}$ is paid as a lump-sum tax which is used to finance a PAYG system. p_t is the transfer per capita of the old generation. Of the remaining part of their income they have to decide how much to save (s_t) for the second period of their life[2]. This gives for the first period consumption c_t^y:

$$c_t^y = 1 - s_t - \frac{p_t}{1+n} \tag{4.2.1}$$

The second period of their life besides the return on their savings they receive a government transfer p_{t+1} financed by the PAYG system. Second-period consumption c_{t+1}^o then is given by:

$$c_{t+1}^o = (1+r)s_t + p_{t+1} \tag{4.2.2}$$

[2]Negative savings are not ruled out. In principle an agent can borrow to achieve negative savings. Since this would imply borrowing against future income it might give an enforcement problem. Especially, if this future income depends on future decisions. However, in this deterministic setting, the level of future income is perfectly predictable. Thus, an enforcement problem is absent.

r is the fixed world interest rate. Utility is assumed to be logarithmic with a private discount factor θ, assumed to be between 0 and 1. The agents optimize:

$$lnc_t^y + \theta lnc_{t+1}^o \tag{4.2.3}$$

subject to eqs. (4.2.1) and (4.2.2). The agents, when deciding on s_t, take the decisions on p_t and p_{t+1} by the government as given. From optimization of eq. (4.2.3) with respect to s_t follows the optimal level of savings s_t^*:

$$s_t^* = \frac{\theta}{1+\theta}\left(1 - \frac{p_t}{1+n}\right) - \frac{1}{1+\theta} \cdot \frac{p_{t+1}}{1+r} \tag{4.2.4}$$

The amount saved s_t^* is a fraction $\frac{\theta}{1+\theta}$ of the net income $1 - \frac{p_t}{1+n}$ minus the present value of the government transfer p_{t+1} to be received in the second period weighted by the private discount factor. A higher value for θ means that more weight is attached to the future which implies that a higher fraction of the net income is saved and the future transfer is less taken into account. More tax p_t to be paid means less is available for savings. A higher transfer p_{t+1} in the next period means less need for private savings.

The government

The decisions on the PAYG system are made in a representative democracy. The government is managed by successive generations of policy makers. Each policy maker is seen as a melting pot of different groups having different, possibly conflicting, interests. These groups are competing among each other for influence on the decisions to be made. The policy adopted reflects the interests of the different groups to the extent they succeed in this. Applied to the present model this implies that every period the government transfer p_t is chosen according to a decision function which is the weighted average of the utilities of the generations present in that period:

$$W_t = \theta lnc_t^o + \lambda(1+n)\left(lnc_t^y + \theta lnc_{t+1}^o\right) \tag{4.2.5}$$

where $\lambda \in [0,\ \infty)$ is the relative political weight of a young individual. The idea of successive policy makers introduces an additional feature. Not only is the present policy maker playing a game with the private sector but it is also playing a game with future policy makers. As the latter game is sequential by construction, Stackelberg behaviour towards future policymakers is a natural assumption. Finally, it is assumed that the economy is dynamically efficient, *i.e.* $r > n$.

4.3 A PAYG system in a representative democracy

When the government is modelled in a representative democracy setting, *i.e.* it is managed by successive generations of policymakers, every period t the policymaker of that period optimizes eq. (4.2.5). Since it acts as a Stackelberg leader towards the private sector, it takes the saving behaviour of the private sector, given by eq. (4.2.4), into account. It also takes the transfers p^*_{t+i}, $i = 1, \ldots$ chosen by future policy makers into account. Given the chosen transfer scheme p^*_t, the optimal level of savings follows from eq. (4.2.4). The solution is given by two first-order linear difference equations[3]. First, the private sector savings s^*_t are given by:

$$s^*_t = \frac{1+n}{r-n} \cdot \frac{\lambda(r-n)-1}{\lambda(1+n)+1} + \frac{\lambda}{\lambda(1+n)+1}(1+r)s^*_{t-1} \tag{4.3.1}$$

For the pension transfer p^*_t the solution reads:

$$p^*_t = \theta\frac{1+n}{r-n} \cdot \frac{1+r}{\lambda(1+\theta)(1+n)+\theta} - \frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r)s^*_{t-1} \tag{4.3.2}$$

Thus, for any initial level of savings, s_0, the evolution of s_t and p_t can be calculated[4] The steady-state values for s^*_t and p^*_t are easily derived from eqs. (4.3.1) and (4.3.2) by setting $s^*_{t-1} = s^*_t$. They are given by $s_{ss} = -\frac{1+n}{r-n}$ and $p_{ss} = \frac{(1+r)(1+n)}{r-n}$. Note that the steady-state values only depend on the population growth rate and the interest rate.

[3]The derivation of the solution is given in the appendix.

[4]Alternatively, as a referee of an earlier version pointed out, one could write these difference equations in terms of p_t and calculate the evolution given an initial transfer level p_0. In that case, the development of p_t becomes independent of θ. This changes some of the results in the sequel but not essentially.

They are independent of the preference parameters λ and θ. The steady-state savings are determined by the maximum amount that can be borrowed, which is given by the net present value of aggregate future incomes[5]. Since it assumed that the economy is dynamically efficient, *i.e.* $r > n$, the steady-state savings of the young are negative. These negative savings are necessary to finance the pension transfer to the present elderly. Negative saving implies borrowing against future resources, in this case the future pension transfer of the present young. The development of the system of savings and pensions can be traced down by rewriting eq. (4.3.1) as follows:

$$s_{ss} - s_t^* = \frac{\lambda(1+r)}{\lambda(1+n)+1}\left(s_{ss} - s_{t-1}^*\right) \tag{4.3.3}$$

If $\frac{\lambda(1+r)}{\lambda(1+n)+1} < 1$, savings decrease monotonically and transfers increase monotonically over time. (The opposite holds if $\frac{\lambda(1+r)}{\lambda(1+n)+1} > 1$.) This condition can be rewritten as $r < n + \frac{1}{\lambda}$. It resembles the Aaron condition (Aaron (1966)). The Aaron condition shows when, for a given level of transfers, the rate of return of the PAYG system is higher than the rate of return of private savings. In an economy which only grows due to population growth, this is equivalent to $r < n$. Thus, in this case, a PAYG system, with positive transfers, can only emerge if the economy is dynamically inefficient. In this chapter, a PAYG system (with a positive transfer from young to elderly) can exist even if the economy is dynamically efficient. The reason is the weight attached to the utility of the current elderly by the policymaker. If their political weight, relative to the young, is sufficiently high, *i.e.* λ is sufficiently low, it holds that $r < n + \frac{1}{\lambda}$. Hence, there are transfers from the current young to the current elderly despite the fact that the economy is dynamically efficient. Moreover, unlike Aaron (1966), transfers are not constant over time. Hence, the rate of return of the PAYG system is given by $\frac{p_{t+1}(1+n)}{p_t}$. Thus, even if no weight is attached to the current elderly, comparing the PAYG system to saving privately implies comparing $\frac{p_{t+1}(1+n)}{p_t}$ to $1+r$.

Figure 4.1 illustrates the evolution of taxes and transfers if $r > n + \frac{1}{\lambda}$ and if $r < n + \frac{1}{\lambda}$. In the latter case, they converge to the steady-state values of $s_{ss} = -\frac{1+n}{r-n}$ and $p_{ss} = \frac{(1+n)(1+r)}{r-n}$. The following proposition summarizes these results:

[5]Future income equals $1+n$, $(1+n)^2$, $(1+n)^3$,.... for period $t+1$, $t+2$, $t+3$,.... Aggregated and discounted back to period t, this amounts to $\frac{1+n}{r-n}$.

Insert Figure 4.1

Proposition 4.1

1. If $r > n + \frac{1}{\lambda}$ *then*

$$s_t = s_t^*, \quad s_t^* > s_{t-1}^*, \quad \lim_{t\to\infty} s_t^* \to \infty$$

$$p_t = p_t^*, \quad p_t^* < p_{t-1}^*, \quad \lim_{t\to\infty} p_t^* \to -\infty$$

2. If $r < n + \frac{1}{\lambda}$ *then*

$$s_t = s_t^*, \quad s_t^* < s_{t-1}^*, \quad \lim_{t\to\infty} s_t^* = s_{ss}$$

$$p_t = p_t^*, \quad p_t^* > p_{t-1}^*, \quad \lim_{t\to\infty} p_t^* = p_{ss}$$

3. If $r = n + \frac{1}{\lambda}$ *then*

$$s_t = s_0$$

$$p_t = p^*(s_0)$$

It is possible to trace down the welfare consequences for the generations alive if a PAYG system is introduced in some period t. The following proposition gives conditions determining the effects of such an introduction on utility of the current generations.

Proposition 4.2 *If the government decides to introduce a PAYG system:*
The current old generation gains in terms of utility iff $r < n + \frac{1}{\lambda}$.
The current young generation gains in terms of utility iff

$$\textbf{A} \quad \frac{1}{r-n} < \lambda < \frac{\theta(r-n)}{(1+n)^2(1+\theta)}$$

or

B $\quad \dfrac{\theta(r-n)}{(1+n)^2(1+\theta)} < \lambda < \dfrac{1}{r-n}$

For the proof, see the appendix. For the current elderly, the effect depends on whether the pension transfers are increasing or decreasing over time. If $r < n + \frac{1}{\lambda}$, the pension transfer is increasing over time. Given that before period t there was no pension transfer, the pension transfer the elderly receive is positive. Thus, they gain in terms of utility with the introduction of a PAYG system. For $r > n + \frac{1}{\lambda}$ the opposite holds. The fact that the savings were fixed for the current elderly at the time the PAYG-scheme was introduced, made it easy to determine the effects of the introduction, it only depended on the pension transfer they received. For the current young, however, the effect depends on the pension transfer when they are old, as well as on the pension transfer for the current old since it is paid by the current young[6]. Moreover, savings are not fixed but depend also on the current and future pension transfer. Consider case **A**. The first inequality with respect to λ implies that the current elderly have to pay a transfer to the current young. This positively affects utility of the current young. This positive effect is not offset by the transfer they have to pay themselves when they are old, if the future transfer is sufficiently low, which is guarenteed by the second inequality. Hence, the current young gain in terms of utility by the introduction of a PAYG system. If λ violates the second inequality, the old-age transfer overcompensates the welfare gain due to the transfer from the current elderly, implying, on total, a loss in terms of utility for the current young. In case λ violates the first inequality, implying a transfer from the current young to the current elderly, which is a utility loss for the current young, this utility loss is not sufficienly compensated by the future transfer the current young receive when they are old. On total, they have a utility loss. A similar story can be told for case **B**.

Stackelberg versus Nash

As noted in the introduction, behavioural assumptions with respect to future decisions are important because future decisions on pension levels are important for the willingness to participate in a PAYG pension scheme now. In this chapter, Stackelberg behaviour is assumed, thus, the effects current policy choices have on future policy choices are

[6]Moreover, because of the assumption of Stackelberg behaviour, *all* future pension transfers are relevant.

explicitely taken into account. The alternative would be to assume Nash behaviour, *i.e.* taking future policy choices as given as done in Verbon and Verhoeven (1992)[7]. A natural question to ask is what difference this assumption makes with respect to the solutions. In case of Nash behaviour, the solution for the savings and the transfers is given by the following two first-order linear difference equations[8]

$$s_t^N = \frac{1+n}{r-n}[\lambda(1+r)-1] + \lambda(1+r)s_{t-1}^N \tag{4.3.4}$$

$$p_t^N = \theta\frac{1+n}{r-n} \cdot \frac{(1+r)[1-\lambda(1+n)]}{\lambda(1+n)+\theta} - \frac{\lambda(1+n)(1+\theta)}{\lambda(1+n)+\theta}(1+r)s_{t-1}^N \tag{4.3.5}$$

where the superscript 'N' refers to the Nash assumption. These solutions clearly differ from the solutions under Stackelberg behaviour given in the previous section. This difference stems from the effect the current pension transfer has on the future pension transfer which in turn affect current savings. This effect is not taken into account when the government takes future policy as given, *i.e.* where Nash behaviour is assumed. It is, however, taken into account in case of Stackelberg behaviour. Straightforward comparison gives the following proposition

Proposition 4.3 *Given any initial level of savings s_0, it holds for all t that*

$$s_t^N > s_t^*, \qquad p_t^N < p_t^*$$

What is the intuition behind this result? As noted before, the current pension transfer affects the future pension transfer which in turn affects current savings. From inserting eq. (4.2.4) into eq. (4.3.2), it can be shown that the effect the current transfer has on the future transfer is positive. This, in turn, negatively affects current savings. Therefore,

[7]This is not the only difference with Verbon and Verhoeven (1992) though. Other differences are: First, Verbon and Verhoeven (1992) exclude negative savings and transfers. Their solution is therefore given by a constrained steady-state with either zero savings or zero transfers. Second, their economy can be dynamically efficient as well as inefficient. But, because of the constraints, this is not really important for their model. The solution to the unconstrained version, however, as analysed here, depends on whether the economy is dynamically efficient or inefficient. In case it is inefficient, the solution is independent of the political parameters. Moreover, consumption becomes zero immediately, *i.e.* directly after the initial period, for both generations.

[8]The derivation is given in the appendix

in case of Nash behaviour, where the positive effect of the current transfer on the future transfer is neglected, this leads to savings which are higher and pension transfers which are lower than under Stackelberg behaviour, where the positive effect of current on future transfers is taken into account.

A social welfare maximizing PAYG system

The solution obtained can be compared to a social welfare maximizing PAYG system which follows from optimization of a Benthamite social welfare function:

$$W_t^{soc} = \sum_{i=0}^{\infty} [\rho(1+n)]^i \left[lnc_i^y + \theta lnc_{i+1}^o\right] \tag{4.3.6}$$

where ρ is the social discount factor. $\tilde{\rho} \equiv \rho(1+n)$ then, denotes the social rate of time preference. This will be used is the rest of this chapter since it simplifies notation. The solution is given by the following two first-order difference equations[9]. For the private sector savings in period t, $s_t^{*,soc}$, it reads:

$$s_t^{*,soc} = \frac{1+n}{r-n} \cdot \frac{r-\tilde{\rho}}{1+\tilde{\rho}} + \frac{1+r}{1+\tilde{\rho}} s_{t-1}^{*,soc} \tag{4.3.7}$$

The solution for the period t pension transfer, $p_t^{*,soc}$, is given by:

$$p_t^{*,soc} = \theta \frac{1+n}{r-n} \cdot \frac{(\tilde{\rho}-n)(1+r)}{1+n+\theta(1+\tilde{\rho})} - \frac{(1+\theta)(1+n)}{1+n+\theta(1+\tilde{\rho})}(1+r)s_{t-1}^{*,soc} \tag{4.3.8}$$

By equating the solution of the social welfare maximizer and the solution of the representative democracy, the following result can be derived:

Proposition 4.4 *Given any initial level of savings s_0, the time paths for taxes and pension transfers chosen by a social welfare maximizer coincide with time paths chosen by a representative democracy if $\tilde{\rho} = n + \frac{1}{\lambda}$.*

Thus, for every value for λ and n, there is a $\tilde{\rho}$ such that the policy chosen by a representative democracy coincides with the policy choice of a social-welfare maximizer and vice

[9] The derivation is given in the appendix.

versa. Both cases take the future into account, but, each of them, in a different way. In the representative democracy, the current government takes the policy choices of future governments into account. If the government is modelled as optimizing a Benthamite social welfare function, the utilities of future generations are taken into account. Remark that in Chapter 3, a similar result was derived. There, however, any policy chosen in a representative democracy can be chosen by a social welfare optimizer but not vice versa. The difference between public debt and public pensions is that the former redistributes between generations, *over* periods while the latter redistributes between generations, *within* periods. Thus, in the former case, the government in the representative democracy has the possibility to shift the burden of taxation over to future, yet unrepresented generations. And, indeed, this will be the case. Hence, policies chosen by a social welfare optimizer that attaches high weight to future generations will never be chosen in a representative democracy.

The following proposition denotes, for a given level of savings s_{t-1}, when generations in period t are better off in a representative democracy than with a social welfare maximizer and vice versa.

Proposition 4.5 *Given s_{t-1}, it holds for:*
The current old generation[10]

$$U_{t-1} < U_{t-1}^{soc} \text{ iff } \tilde{\rho} < n + \frac{1}{\lambda}$$

The current young generation

$$\textbf{A} \quad U_t < U_t^{soc} \text{ iff } n + \frac{1}{\lambda} < \tilde{\rho} < n + \lambda(1+n)^2 \frac{1+\theta}{\theta}$$

$$\textbf{B} \quad U_t < U_t^{soc} \text{ iff } n + \lambda(1+n)^2 \frac{1+\theta}{\theta} < \tilde{\rho} < n + \frac{1}{\lambda}$$

Proof: see the appendix. For the current elderly, the result is straightforward. If $\tilde{\rho}$ is smaller than $n + \frac{1}{\lambda}$, the pension transfer in the representative democracy is smaller than the pension transfer of a social welfare optimizer. Hence, given their level of savings, their utility is lower in the representative democracy. For the current young results are more complicated. The effect on their utility depends on the current as well as on

[10]The superscript 'soc' refers to social welfare optimization.

all future pension transfers. Moreover, unlike the current elderly, their savings are not fixed but also vary with the present and future pension transfer. Consider case **A**. If $n + \frac{1}{\lambda} < \tilde{\rho}$, the pension transfer with the social welfare optimizer is smaller than the transfer in the representative democracy. This implies a gain for the current young in terms of utility with the social welfare optimizer when they are young, but a loss when they are old. This loss is, however, not sufficient to compensate for the gain incurred when they were young. Therefore, on total they gain in terms of utility with the social welfare optimizer relative to the representative democracy. If $\tilde{\rho} < n + \frac{1}{\lambda}$, the utility loss incurred when young due to the higher transfer paid under the social welfare optimizer relative to the representative democracy is not sufficiently compensated in terms of utility by the higher transfer received when they are old. Hence, utility is higher in the representative democracy. A similar story can be told if $\tilde{\rho} > n + \lambda(1+n)^2 \frac{1+\theta}{\theta}$. In that case, the utility gain due to the higher transfer received in their old-age by the current young from the social welfare optimizer is not sufficient to compensate for the utility loss due to the higher transfer paid to the current elderly. Hence, utility is lower with the social welfare optimizer than in the representative democracy. A similar story can be told for case **B**. Furthermore, note that if both generations get higher utility in the representative democracy than with the social welfare optimizer, this must imply that some future generations get lower utility in the representative democracy. Otherwise, the social welfare optimizer could improve in terms of utility by choosing the same policy as in the representative democracy.

4.4 The effects of parameter changes

In reality, the exogenous parameters in this model are not constant over time. Population growth rates, interest rates and political and private preference parameters (λ and θ respectively) change over time. This section analyses the effects of changes in the exogenous parameters. The focus will be on unanticipated changes. Note that, the length of one period corresponds to one generation, *i.e.*, about 30 years. Hence, people anticipating on changes in the political balance, the private discount factor or the interest rate taking place in the next stage of their life is rather unrealistic. Changes in the population growth rate are, more or less, forseeable over such a period of time. Therefore anticipated as well as unanticipated shocks are analysed in this case.

The reference situation in the sequel will be $\frac{\lambda(1+r)}{\lambda(1+n)+1} < 1$, *i.e.* decreasing saving rates and increasing transfers, and a positive s_0. Period $\tilde{t}$ denotes the period the parameter change occurs. A subscript '1' denotes the value of a parameter before the change, a subscript '2' denotes its value after the change. The mathematical derivations of the results are given in the appendix.

The effect of an increase in political power of an old individual An increase in the political power of an old individual is reflected by a decrease in λ. If the change is unanticipated, the young of period $\tilde{t}-1$ expect the pension policy not to change. Their savings decision is based on this expectation. When they are confronted with their increased political weight the following period, they will use it to raise their pension benefit. However, since they did not foresee this possibility the period before, their savings are 'too high'. This partly offsets the increase in pensions. In other words, though their increased political power gives the elderly the possibility to force a higher pension benefit, their lower 'need' for it partly offsets the increase. The total effect is positive. Note that the steady-state levels of savings and pensions are unaffected. If the change in λ is large enough, a regime switch may occur from decreasing savings and increasing transfers to increasing savings and decreasing transfers. The following proposition summarizes:

Proposition 4.6 *In case of an unanticipated increase in λ in some period $\tilde{t}$, the initial effects are*

$$p_{\tilde{t}}(\textit{with change}) > p_{\tilde{t}}(\textit{no change})$$

$$s_{\tilde{t}}(\textit{with change}) < s_{\tilde{t}}(\textit{no change})$$

Steady-state levels are unaffected.

The effect of an increase in the private discount factor An increase in the private discount factor at some date $\tilde{t}$ implies a higher weight for old-age utility of the generation $\tilde{t}$. Ceteris paribus, this implies that they will increase their savings. However, savings also depend on current and future pension benefits. The government, confronted with a generation of elderly with a relatively low private discount factor and a generation of young with a higher private discount factor, will decrease the pension benefit for the current elderly. This lower pension benefit for the current elderly positively affects the savings of the current young, ceteris paribus. However, savings also depend on the pension benefit the current young expect to receive themselves. Ceteris paribus, a higher weight for old-age consumption implies a higher pension benefit. Since this negatively affects savings whereas the lower pension benefit for the current elderly positively affects savings, the total effect is ambiguous and depends on the relation between λ, n, r and the change in θ. From the condition given in Proposition 4.7, the following can be inferred. If λ or r is relatively high, the initial effect on savings is more likely to be positive. In the first case, a relatively high λ implies relatively more political power for the young. Therefore, current young know that for their old age they cannot rely too much on their pension benefit. Hence, they better save more for themselves. In the second case, a relatively high r, saving privately is relatively more attractive than the PAYG system. The effect of n on the initial jump of savings is ambiguous because n has two, opposing, effects. On the one hand a relatively high n implies relatively more weight for the young. On the other hand, it implies that the PAYG system is relatively more attractive than saving privately. The savings of generation $\tilde{t}+1$ and subsequent generations are only affected by the effect on the savings of generation $\tilde{t}$[11]. Since the level of $s_{\tilde{t}}$ is ambiguous, the level of $s_{\tilde{t}-1}$ and so forth, is ambiguous as well[12]. The effect on the pension level of period $\tilde{t}+1$ and subsequent periods, depends in a similar way on $s_{\tilde{t}}$. This follows immediately from eq. (4.3.2). Finally, note that the steady-state levels of pensions and savings are unaffected. The following proposition summarizes:

Proposition 4.7 *In case of an unanticipated increase in θ in some period $\tilde{t}$, the initial effects are*

$$p_{\tilde{t}}(\textit{with change}) < p_{\tilde{t}}(\textit{no change})$$

$$s_{\tilde{t}}(\textit{with change}) > (<)\; s_{\tilde{t}}(\textit{no change}) \textit{ iff } \lambda(r-n) + \frac{1+r(1+n)}{r-n} > (<) \frac{\theta_1}{1+\theta_2}$$

Steady-state levels are unaffected.

[11]This follows from eq. (4.3.1), which does not depend on θ.

[12]In principle, it is possible to calculate a similar condition as done for $s_{\tilde{t}}$. However, since it requires calculation back until period $\tilde{t}-1$, the last period before the change, these conditions become increasingly complex without providing further insight. Therefore, they are omitted.

The effect of an increase in the interest rate An increase in the interest rate makes private saving more attractive relative to the PAYG system. Therefore, the initial effect on private savings is positive and the effect on pensions is negative. The effects on the steady-state levels are similar. The new steady-state level of pensions is lower, the new steady-state level of savings is higher. Furthermore, since $\frac{\lambda(1+r)}{\lambda(1+n)+1}$ is increasing in r, the change in r may cause a regime switch, *i.e.* from decreasing savings and increasing pension benefits to increasing savings and decreasing pension benefits. The following proposition summarizes:

Proposition 4.8 *In case of an unanticipated increase in r in some period $\tilde{t}$, the initial effects are*

$$p_{\tilde{t}}(\textit{with change}) < p_{\tilde{t}}(\textit{no change})$$

$$s_{\tilde{t}}(\textit{with change}) > s_{\tilde{t}}(\textit{no change})$$

Steady-state levels change

$$p_{ss}^{new} < p_{ss}^{old} \qquad s_{ss}^{new} > s_{ss}^{old}$$

The effect of a decrease in the growth rate of the population From a policy perspective, a change in the population growth rate is most interesting one. In this case, anticipated as well as unanticipated changes are analysed. In either case, a decrease in the population growth rate makes the PAYG system less attractive compared to private savings. This can be inferred from the decrease of the steady-state pension benefit and the increase in the steady-state savings level. The decrease in population growth has two, opposing, effects on the pension transfer. On the one hand, there is a political effect. The extra number of elderly, relative to the number of young, creates an upward pressure on the pension level. On the other hand, relatively more elderly increases the cost for the current young if the pension transfer would remain the same (the cost effect). This creates a downward pressure on the pension transfer.

If there is an unanticipated decrease of the population growth rate, the initial effect depends on the savings level of the current elderly, $s_{\tilde{t}-1}$. If this is relatively low, the pension level decreases. Otherwise, it increases. What is the intuition? A relatively low $s_{\tilde{t}-1}$ implies that the current pension transfer is relatively high. Maintaining the current pension transfer is, therefore, relatively costly. Hence, the cost effect will dominate the political effect and a decrease in the pension transfer and an increase in the savings level can be observed. If $s_{\tilde{t}-1}$ is relatively high, the opposite holds. Note from Proposition 4.9 that there are levels of $s_{\tilde{t}-1}$ for which both $p_{\tilde{t}}$ as well as $s_{\tilde{t}}$ decrease. A decrease in $p_{\tilde{t}}$ positively affects $s_{\tilde{t}}$, as can be deduced from eq. (4.2.4). However, if this decrease is sufficiently small, this decrease is offset by the decrease in n since it is the ratio of $p_{\tilde{t}}$ over $1+n$ which matters (see eq. (4.2.4)). Furthermore, note that this implies that if private savings are relatively high, if initially no PAYG system exists, it may come about even if the growth rate of the population decreases[13].

In case of an anticipated change, the initial effect of a decrease in the population growth rate on the pension level is negative, *i.e.* at the time the change in n becomes known, there is a downward jump in the pension level and an upward jump of savings. The reason is that, in this case, the cost effect is taken into account directly whereas the political effect is only indirectly taken into account via $p_{\tilde{t}+1}$. The direct political effect is absent until the time of the change implying still a relatively high weight for the current young. Hence, savings increase and the pension benefit to the elderly in period decreases. The following proposition summarizes:

Proposition 4.9 *In case of an unanticipated decrease in n in some period $\tilde{t}$, the initial effects are*

$$p_{\tilde{t}}(\textit{with change}) > (<)\ p_{\tilde{t}}(\textit{no change})$$

$$\textit{iff } s_{\tilde{t}-1} > (<) -\frac{1+n_1}{r-n_1}\left(1-\frac{\lambda(1+\theta)(1+n_1)+\theta}{\lambda(1+\theta)(r-n_2)}\right)$$

$$s_{\tilde{t}}(\textit{with change}) < (>)\ s_{\tilde{t}}(\textit{no change})$$

$$\textit{iff } s_{\tilde{t}-1} > (<) -\frac{1+n_1}{r-n_1}\left(1-\frac{\lambda(1+n_1)+1}{\lambda(r-n_2)}\right)$$

In case of an anticipated decrease in n in some period $\tilde{t}$, the initial effects in period $\tilde{t}-j$, when the future shock becomes known, are

$$p_{\tilde{t}-j}(\textit{with change}) < p_{\tilde{t}-j}(\textit{no change})$$

[13] A similar result is derived in Meijdam and Verbon (1995).

$$s_{\bar{t}-j}(with\ change) > s_{\bar{t}-j}(no\ change)$$

Steady-state levels change

$$p_{ss}^{new} < p_{ss}^{old} \qquad s_{ss}^{new} > s_{ss}^{old}$$

4.5 Summary and concluding remarks

The intention of this chapter is to provide additional insights into the evolution of PAYG public pension systems or, more general, social security systems. From a normative point of view, social security programs might be installed because they lead to a Pareto superior allocation of endowments. This, however, can only be the case if $r < n$, *i.e.* the economy is dynamically inefficient. In positive models, social security systems can arise if the economy is dynamically efficient. The existing literature up to date concerning positive explanations is mainly based on median voter models or assumptions of altruism. This chapter does not use a median voter framework nor does it rely on the assumption of altruism. Instead, the pension level is chosen in a representative democracy.

An important feature of this chapter is the notion of Stackelberg behaviour. Not only is the government assumed to be a Stackelberg leader towards the private sector, taking account of the behavioural responses of the private sector, but it also acts as a Stackelberg leader towards future governments. Because of this assumption, the present young anticipate on the behavioural responses of future generations which implies that they are able to use their savings strategically in a similar way as in Veall (1986).

The following results were derived: First, even without the assumption of altruism a PAYG pension system might arise. The reason is that agents rationally anticipate future decisions and, besides that, both generations have political influence on the decisions taken. In addition to the latter point, one might say that the political influence of the elder generation replaces the assumption of altruism. Whether a PAYG system with positive transfers from young to elderly indeed comes into existence depends on the parameters of the model. Conditions were derived that determine whether a PAYG pension system will emerge or the old generation will be forced to make transfers to the young. These conditions were a function of the population growth rate, the interest rate and the political weight of a young individual relative to an elderly. Also, the welfare consequences for the current generations were analysed if the government decides to introduce a PAYG system.

Next, the difference between the assumption of Nash behaviour and of Stackelberg behaviour was analysed. Compared to the solution with the assumption of Nash behaviour, the level of savings was lower if the effect of current pension transfers on future transfers was taken into account. The current transfer postively affected the future transfer, which, in turn, negatively affects current savings. This, however, was neglected in case of Nash behaviour. Savings are therefore higher with Nash behaviour than with Stackelberg behaviour. For the level of pension transfers the opposite held.

Furthermore, the PAYG system chosen in a representative democracy was compared to the policy choice of social welfare maximizer. It appeared that every policy chosen by a social welfare maximizer is feasible in a representative democracy as well and vice versa. The reason was that both cases took the future into account, but, each of them, in a different way. In the representative democracy, the current government took the policy choices of future governments into account. If the government is modelled as optimizing a Benthamite social welfare function, the utilities of future generations were taken into account. In addition, a welfare comparison was made between these two cases.

Finally, the effects of changes in the exogenous parameters were analysed. An increase in the political power of an old individual led initially to an upward jump in the pension benefit, and a downward jump in the private savings level. The steady-state levels were unaffected because they were only determined by parameters related to total discounted income. In case individual consumers became less impatient and attached more weight to old-age consumption, reflected by a higher private discount factor, the initial effect depended on the other exogenous parameters. The steady-state levels were, again, unaffected. An increase in the interest rate made saving privately more attractive than the PAYG system as a means to transfer wealth to the next period of life. Hence, pensions jumped downward initially while savings jumped upward. The steady-state levels changed in the same direction. From a policy perspective, in the face of the ageing population in many western countries, the change in the population growth rate was the most interesting one. In case of an unforeseen decrease in the population growth rate, the initial effects depended on the level of savings of the elderly when the change occurred. If these were relatively high, the pension level would jump upward, while private savings would jump downward. A decrease in the population growth rate had two, opposing effects. On the one hand, there was a political effect because of the increase in the number of elderly relative to the number of young. On the other hand, there was a cost effect because keeping the pension at its current level became more costly for the current young. In case of a relatively low savings level, the latter effect dominated, causing a decline in the pension level and an increase in the savings level. If savings were relatively high, the opposite held. In case a future decrease in the growth rate of the population was anticipated, the pension level jumped downward whereas private savings jumped upward when the future change became known.

A Appendix

A.1 The derivation of the PAYG-scheme in a representative democracy

The solution to the infinite horizon model is obtained by first solving the finite horizon model and, then, taking the limit to obtain the solution to the infinite horizon model[14]. Assume that there is some final period T. Then, $s_T = p_{T+1} = 0$. The government in period T then optimizes:

$$\max_{p_T} \theta ln[(1+r)s_{T-1} + p_T] + \lambda(1+n)ln[1 - \frac{p_T}{1+n}] \tag{A-1}$$

Straightforward optimization shows that

$$p_T = \theta \frac{1+n}{\lambda(1+n)+\theta} + \frac{\lambda(1+n)}{\lambda(1+n)+\theta}(1+r)s_{T-1} \tag{A-2}$$

Next, the government in period $T-1$ optimizes

$$\max_{p_{T-1}} \theta ln[(1+r)s_{T-2} + p_{T-1}] +$$

[14]Note that standard dynamic programming techniques are not applicable in this case because of the form of the objective function.

$$+\lambda(1+n)\left\{ln[1-s_{T-1}-\frac{p_{T-1}}{1+n}]+\theta ln[(1+r)s_{T-1}+p_T]\right\} \tag{A-3}$$

Since the government is Stackelberg leader with respect to the private sector, it takes eq. (4.2.4), the savings behaviour of the private sector, into account. Using this equation in eq. (A-3), it can rewritten as:

$$\max_{p_{T-1}} \theta ln[(1+r)s_{T-2}+p_{T-1}]+\lambda(1+n)(1+\theta)ln[1-\frac{p_{T-1}}{1+n}+\frac{p_T}{1+r}] \tag{A-4}$$

Each government is also Stackelberg leader with respect to future governments. Hence, using eq. (4.2.4), p_T can be rewritten as a function of p_{T-1}. Inserting this in eq. (A-4) and optimizing gives:

$$p_{T-1}=\theta\frac{1+n}{1+r}\cdot\frac{(1+r)+(1+n)}{\lambda(1+\theta)(1+n)+\theta}-\frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r)s_{T-2} \tag{A-5}$$

Doing this repeatedly gives

$$p_k=\theta\frac{1+n}{r-n}\cdot\frac{1+r}{\lambda(1+\theta)(1+n)+\theta}\left[1-\left(\frac{1+n}{1+r}\right)^{T-k+1}\right]-$$

$$-\frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r)s_{k-1} \tag{A-6}$$

for $k=0,\ldots,T-1$. Eq. (4.3.2) then follows from $T\rightarrow\infty$. Eq. (4.3.1) follows from inserting eq. (4.3.2) into eq. (4.2.4).

A.2 Proof of Proposition 4.2

Given the logarithmic utility function, it is sufficient to compare c_t^o for the current elderly and $c_t^y(c_{t+1}^o)^\theta$ for the current young.

From eq. (4.2.4) follows that if there was no PAYG pension system savings s_{t-1} are equal to $\frac{\theta}{1+\theta}$. Hence, in that case $c_t^o=(1+r)\frac{\theta}{1+\theta}$. If the government introduces a PAYG pension system, c_t^o becomes

$$c_t^o=\frac{\theta(1+r)(1+n)}{[\lambda(1+\theta)(1+n)+\theta](r-n)}+\frac{\theta(1+r)}{[\lambda(1+\theta)(1+n)+\theta]}\frac{\theta}{1+\theta}$$

Direct comparison shows that consumption of the current elderly is higher (lower) by the introduction of the PAYG pension system iff

$$\lambda(r-n) < (>)\ 1$$

Using eqs. (4.2.1), (4.2.2) gives for $c_t^y (c_{t+1}^o)^\theta$ in case there is no PAYG pension system

$$\left(\frac{1}{1+\theta}\right)^{1+\theta} [\theta(1+r)]^\theta$$

Using eqs. (4.2.1), (4.2.2) and eqs. (4.3.1), (4.3.2) gives for $c_t^y (c_{t+1}^o)^\theta$ in case there is a PAYG pension system

$$\left[\frac{\lambda(1+r)[(1+n)+(r-n)\frac{\theta}{1+\theta}]}{(r-n)[\lambda(1+n)+1][\lambda(1+\theta)(1+n)+\theta]}\right]^{1+\theta} [\theta(1+r)]^\theta$$

Direct comparison shows that utility is higher (lower) with the PAYG pension system iff

$$[\lambda(r-n)-1][\lambda(1+n)^2(1+\theta)-\theta(r-n)] < (>) 0$$

Q.E.D.

A.3 Nash

The optimal level of private savings is given by eq. (4.2.4):

$$s_t = \frac{\theta}{1+\theta}\left(1-\frac{p_t}{1+n}\right) - \frac{1}{1+\theta}\cdot\frac{p_{t+1}}{1+r} \tag{A-7}$$

The first-order condition for the government is given by

$$\frac{\theta}{c_t^o} = \frac{\lambda}{c_t^y}$$

Hence, the optimal transfer level is given by

$$p_t = \frac{\theta(1-s_t)-\lambda(1+r)s_{t-1}}{\lambda+\frac{\theta}{1+n}} \tag{A-8}$$

Using eqs. (A-7) and (A-8) the following dynamical system can be derived:

$$\begin{bmatrix} s_t \\ \\ p_{t+1} \end{bmatrix} = M \begin{bmatrix} s_{t-1} \\ \\ p_t \end{bmatrix} + N \tag{A-9}$$

where

$$M = \begin{bmatrix} -\frac{\lambda}{\theta}(1+r) & -\left(\frac{\lambda}{\theta} + \frac{1}{1+n}\right) \\ (1+\theta)\frac{\lambda}{\theta}(1+r)^2 & (1+r)\left[(1+\theta)\frac{\lambda}{\theta} + \frac{1}{1+n}\right] \end{bmatrix} \qquad N = \begin{bmatrix} 1 \\ -(1+r) \end{bmatrix}$$

The following two eigenvalues of M can be calculated

$$\mu_1 = \lambda(1+r), \qquad \mu_2 = \frac{1+r}{1+n}$$

Since the economy is assumed to be dynamically efficient, *i.e.* $r > n$, it holds that $\mu_2 > 1$. For the dynamical system to be saddlepoint stable, it must hold that $\lambda(1+r) < 1$[15]. Then, applying the result of Blanchard and Kahn (1980), eqs. (4.3.4) and (4.3.5) are derived.

A.4 The derivation of the social welfare maximizing PAYG system

The solution to the infinite horizon model is obtained by first solving the finite horizon model and, then, taking the limit to obtain the solution to the infinite horizon model[16]. Assume that there is some final period T. Then, $s_T = p_{T+1} = 0$. In the final period, the following optimization problem results:

$$\max_{p_T} \left(\frac{1+n}{1+\tilde{\rho}}\right)^{T-1} \theta ln[(1+r)s_{T-1} + p_T] + \left(\frac{1+n}{1+\tilde{\rho}}\right)^{T} ln[1 - \frac{p_T}{1+n}] \tag{A-10}$$

Here the social rate of time preference $\tilde{\rho}$ instead of the social discount factor ρ has been used since it simplifies notation. Note that $\rho = \frac{1}{1+\tilde{\rho}}$. Straightforward optimization shows that:

$$p_T = \theta\frac{(1+\tilde{\rho})(1+n)}{\theta(1+\tilde{\rho}) + (1+n)} - \frac{(1+n)}{\theta(1+\tilde{\rho}) + (1+n)}(1+r)s_{T-1} \tag{A-11}$$

Next, the optimization problem in period $T-1$ is given by:

[15] If $\lambda(1+r) > 1$, an unstable system would result.

[16] Note that standard dynamic programming techniques are not applicable in this case because of the form of the objective function.

$$\max_{p_{T-1}} \left(\frac{1+n}{1+\tilde{\rho}}\right)^{T-2} \theta ln[(1+r)s_{T-2}+p_{T-1}]+$$

$$+\left(\frac{1+n}{1+\tilde{\rho}}\right)^{T-1} \left\{ln[1-s_{T-1}-\frac{p_{T-1}}{1+n}]+\theta ln[(1+r)s_{T-1}+p_T]\right\}+ \tag{A-12}$$

$$+\left(\frac{1+n}{1+\tilde{\rho}}\right)^{T} ln[1-\frac{p_T}{1+n}]$$

Since the social welfare maximizer is Stackelberg leader with respect to the private sector, it takes the behaviour of the private sector, as given by eq. (4.2.4), into account. Then, using eqs. (4.2.4) and (A-11), eq. (A-12) can be rewritten as:

$$\max_{p_{T-1}} \theta ln[(1+r)s_{T-2}+p_{T-1}]+\frac{1+n}{1+\tilde{\rho}}\left(1+\theta+\frac{1+n}{1+\tilde{\rho}}\right) ln[1+\frac{1+n}{1+r}-\frac{p_{T-1}}{1+n}] \tag{A-13}$$

Again, straightforward optimization shows:

$$p_{T-1}=\theta\frac{(1+\tilde{\rho})^2(1+n)}{(1+\tilde{\rho})(1+n)+(1+n)^2+\theta[(1+\tilde{\rho})(1+n)+(1+\tilde{\rho})^2]}\left(1+\frac{1+n}{1+r}\right)-$$

$$-\frac{(1+n)[(1+\theta)(1+\tilde{\rho})+(1+n)]}{(1+\tilde{\rho})(1+n)+(1+n)^2+\theta[(1+\tilde{\rho})(1+n)+(1+\tilde{\rho})^2]}(1+r)s_{T-2} \tag{A-14}$$

Doing this repeatedly gives:

$$p_k=\theta\frac{\frac{1+r}{r-n}\left[1-\left(\frac{1+n}{1+r}\right)^{T-k+1}\right](1+\tilde{\rho})}{\frac{1}{\tilde{\rho}-n}\left[(1+\theta)(1+\tilde{\rho})-[(1+\theta)(1+\tilde{\rho})-(\tilde{\rho}-n)]\left(\frac{1+n}{1+\tilde{\rho}}\right)^{T-k}\right]+\frac{\theta}{1+n}}-$$

$$-\frac{\frac{1+r}{\tilde{\rho}-n}\left[(1+\theta)(1+\tilde{\rho})-[(1+\theta)(1+\tilde{\rho})-(\tilde{\rho}-n)]\left(\frac{1+n}{1+\tilde{\rho}}\right)^{T-k}\right]}{\frac{1}{\tilde{\rho}-n}\left[(1+\theta)(1+\tilde{\rho})-[(1+\theta)(1+\tilde{\rho})-(\tilde{\rho}-n)]\left(\frac{1+n}{1+\tilde{\rho}}\right)^{T-k}\right]+\frac{\theta}{1+n}}s_{k-1} \tag{A-15}$$

for $k=0,\ldots,T-1$. Eq. (4.3.8) then follows from $T\rightarrow\infty$. Eq. (4.3.7) follows from inserting eq. (4.3.8) into eq. (4.2.4).

A.5 Proof of Proposition 4.5

Given the logarithmic utility function, it is sufficient to compare c_t^o for the current elderly and $c_t^y(c_{t+1}^o)^\theta$ for the current young.
Using eq. (4.2.2) and eqs. (4.3.2) and (4.3.8) to obtain c_t^o and $c_t^{o,soc}$ respectively:

$$c_t^o = \frac{\theta(1+r)(1+n)}{[\lambda(1+\theta)(1+n)+\theta](r-n)} + \frac{\theta(1+r)}{[\lambda(1+\theta)(1+n)+\theta]}s_{t-1}$$

$$c_t^{o,soc} = \frac{\theta(\tilde{\rho}-n)(1+r)(1+n)}{[(1+n)+\theta(1+\tilde{\rho})](r-n)} + \frac{\theta(\tilde{\rho}-n)(1+r)}{[(1+n)+\theta(1+\tilde{\rho})]}s_{t-1}$$

Direct comparison shows that

$$c_t^{o,soc} > (<)\ c_t^o \Leftrightarrow \tilde{\rho} > (<)\ n + \frac{1}{\lambda}$$

Using eqs. (4.2.1), (4.2.2) and eqs. (4.3.1), (4.3.2) and eqs. (4.3.7), (4.3.8) to obtain $c_t^y(c_{t+1}^o)^\theta$ and $c_t^{y,soc}(c_{t+1}^{o,soc})^\theta$ respectively:

$$c_t^y(c_{t+1}^o)^\theta = \left[\frac{\lambda(1+r)[(1+n)+(r-n)s_{t-1}]}{(r-n)[\lambda(1+n)+1][\lambda(1+\theta)(1+n)+\theta]}\right]^{1+\theta}[\theta(1+r)]^\theta$$

$$c_t^{y,soc}(c_{t+1}^{o,soc})^\theta = \left[\frac{(1+r)(\tilde{\rho}-n)[(1+n)+(r-n)s_{t-1}]}{(1+\tilde{\rho})(r-n)[(1+n)+\theta(1+\tilde{\rho})]}\right]^{1+\theta}[\theta(1+r)]^\theta$$

Direct comparison shows that

$$c_t^y(c_{t+1}^o)^\theta > (<)\ c_t^{y,soc}(c_{t+1}^{o,soc})^\theta \Leftrightarrow$$

$$\Leftrightarrow (\tilde{\rho} - [n + \frac{1}{\lambda}])(\tilde{\rho} - [n + \lambda(1+n)^2\frac{1+\theta}{\theta}) > (<)\ 0$$

Q.E.D.

A.6 Parameter changes

In the sequel, a subscript '1' denotes the value of a parameter before the change occurs, *i.e.* in the periods $1, \ldots, \tilde{t}$. The subscript '2' denotes the value of a parameter after the change, *i.e.* periods $\tilde{t}, \ldots$.

The effect of an increase in political power of an old individual

The optimization problem in period $\tilde{t}$ is given by

$$\max_{p_{\tilde{t}}} \theta ln[(1+r)s_{\tilde{t}-1}+p_{\tilde{t}}]+$$
$$+\lambda_2(1+n)\left\{ln[1-s_{\tilde{t}}-\frac{p_{\tilde{t}}}{1+n}]+\theta ln[(1+r)s_{\tilde{t}}+p_{\tilde{t}+1}]\right\} \qquad \text{(A-16)}$$

subject to

$$p_{\tilde{t}+1}=\theta\frac{1+n}{r-n}\cdot\frac{1+r}{\lambda_2(1+\theta)(1+n)+\theta}-\frac{\lambda_2(1+\theta)(1+n)}{\lambda_2(1+\theta)(1+n)+\theta}(1+r)s_{\tilde{t}} \qquad \text{(A-17)}$$

$$s_{\tilde{t}}=\frac{\theta}{1+\theta}\left(1-\frac{p_{\tilde{t}}}{1+n}\right)-\frac{1}{1+\theta}\cdot\frac{p_{\tilde{t}+1}}{1+r} \qquad \text{(A-18)}$$

Straightforward optimization shows that

$$p_{\tilde{t}}=\theta\frac{1+n}{r-n}\cdot\frac{1+r}{\lambda_2(1+\theta)(1+n)+\theta}-\frac{\lambda_2(1+\theta)(1+n)}{\lambda_2(1+\theta)(1+n)+\theta}(1+r)s_{\tilde{t}-1} \qquad \text{(A-19)}$$

In period $\tilde{t}-1$, policy is chosen assuming that λ is equal to λ_1 in period $\tilde{t}$. Hence, the optimization problem is given by:

$$\max_{p_{\tilde{t}-1}} \theta ln[(1+r)s_{\tilde{t}-2}+p_{\tilde{t}-1}]+$$
$$+\lambda_1(1+n)\left\{ln[1-s_{\tilde{t}-1}-\frac{p_{\tilde{t}-1}}{1+n}]+\theta ln[(1+r)s_{\tilde{t}-1}+p_{\tilde{t}}]\right\} \qquad \text{(A-20)}$$

subject to

$$p_{\tilde{t}}=\theta\frac{1+n}{r-n}\cdot\frac{1+r}{\lambda_2(1+\theta)(1+n)+\theta}-\frac{\lambda_2(1+\theta)(1+n)}{\lambda_2(1+\theta)(1+n)+\theta}(1+r)s_{\tilde{t}-1} \qquad \text{(A-21)}$$

$$s_{\tilde{t}-1}=\frac{\theta}{1+\theta}\left(1-\frac{p_{\tilde{t}-1}}{1+n}\right)-\frac{1}{1+\theta}\cdot\frac{p_{\tilde{t}}}{1+r} \qquad \text{(A-22)}$$

Straightforward optimization shows that

$$p_{\tilde{t}-1} = \theta \frac{1+n}{r-n} \cdot \frac{1+r}{\lambda_1(1+\theta)(1+n)+\theta} - \frac{\lambda_1(1+\theta)(1+n)}{\lambda_1(1+\theta)(1+n)+\theta}(1+r)s_{\tilde{t}-2} \quad \text{(A-23)}$$

The initial jump in the pension benefit follows from comparison of eq. (A-19) with the level of $p_{\tilde{t}}$ if there was no change in λ. Straightforward computation shows that $p_{\tilde{t}}$(no change) $<$ $p_{\tilde{t}}$(with change). In a similar way it follows that $s_{\tilde{t}}$(no change) $>$ $s_{\tilde{t}}$(with change).

The effect of an increase in the private discount factor

The optimization problem in period $\tilde{t}$ is given by

$$\max_{p_{\tilde{t}}} \theta_1 ln[(1+r)s_{\tilde{t}-1} + p_{\tilde{t}}] +$$

$$+ \lambda(1+n)\left\{ ln[1 - s_{\tilde{t}} - \frac{p_{\tilde{t}}}{1+n}] + \theta_2 ln[(1+r)s_{\tilde{t}} + p_{\tilde{t}+1}]\right\} \quad \text{(A-24)}$$

subject to

$$p_{\tilde{t}+1} = \theta_2 \frac{1+n}{r-n} \cdot \frac{1+r}{\lambda(1+\theta_2)(1+n)+\theta} - \frac{\lambda(1+\theta_2)(1+n)}{\lambda(1+\theta_2)(1+n)+\theta_2}(1+r)s_{\tilde{t}} \quad \text{(A-25)}$$

$$s_{\tilde{t}} = \frac{\theta_2}{1+\theta_2}\left(1 - \frac{p_{\tilde{t}}}{1+n}\right) - \frac{1}{1+\theta_2} \cdot \frac{p_{\tilde{t}+1}}{1+r} \quad \text{(A-26)}$$

Straightforward optimization shows that

$$p_{\tilde{t}} = \theta_1 \frac{1+n}{r-n} \cdot \frac{1+r}{\lambda(1+\theta_2)(1+n)+\theta_1} - \frac{\lambda(1+\theta_2)(1+n)}{\lambda(1+\theta_2)(1+n)+\theta_1}(1+r)s_{\tilde{t}-1} \quad \text{(A-27)}$$

In period $\tilde{t}-1$, policy is chosen assuming that θ remains equal to θ_1 in period $\tilde{t}$. Hence, the optimization problem is given by:

$$\max_{p_{\tilde{t}-1}} \theta_1 ln[(1+r)s_{\tilde{t}-2} + p_{\tilde{t}-1}] +$$

$$+ \lambda_1(1+n)\left\{ ln[1 - s_{\tilde{t}-1} - \frac{p_{\tilde{t}-1}}{1+n}] + \theta_1 ln[(1+r)s_{\tilde{t}-1} + p_{\tilde{t}}]\right\} \quad \text{(A-28)}$$

subject to

$$p_{\tilde{t}} = \theta_1 \frac{1+n}{r-n} \cdot \frac{1+r}{\lambda_2(1+\theta_1)(1+n)+\theta_1} - \frac{\lambda(1+\theta_1)(1+n)}{\lambda(1+\theta_1)(1+n)+\theta_1}(1+r)s_{\tilde{t}-1} \quad \text{(A-29)}$$

$$s_{\tilde{t}-1} = \frac{\theta_1}{1+\theta_1}\left(1 - \frac{p_{\tilde{t}-1}}{1+n}\right) - \frac{1}{1+\theta_1} \cdot \frac{p_{\tilde{t}}}{1+r} \quad \text{(A-30)}$$

Straightforward optimization shows that

$$p_{\tilde{t}-1} = \theta_1 \frac{1+n}{r-n} \cdot \frac{1+r}{\lambda_1(1+\theta_1)(1+n)+\theta_1} - \frac{\lambda(1+\theta_1)(1+n)}{\lambda(1+\theta_1)(1+n)+\theta_1}(1+r)s_{\tilde{t}-2} \quad \text{(A-31)}$$

The initial jump in the pension benefit follows from comparison of eq. (A-27) with the level of $p_{\tilde{t}}$ if there was no change in θ. Straightforward computation shows that $p_{\tilde{t}}$(no change) $>$ $p_{\tilde{t}}$(with change). In a similar way it follows that $s_{\tilde{t}}$(no change) $<$ ($>$) $s_{\tilde{t}}$(with change) iff $\lambda(r-n) + \frac{1+r(1+n)}{r-n} > (<) \frac{\theta_1}{1+\theta_2}$.

The effect of an increase in the interest rate

The optimization problem in period $\tilde{t}$ is given by

$$\max_{p_{\tilde{t}}} \theta ln[(1+r_2)s_{\tilde{t}-1} + p_{\tilde{t}}] +$$

$$+ \lambda(1+n)\left\{ln[1 - s_{\tilde{t}} - \frac{p_{\tilde{t}}}{1+n}] + \theta ln[(1+r_2)s_{\tilde{t}} + p_{\tilde{t}+1}]\right\} \quad \text{(A-32)}$$

subject to

$$p_{\tilde{t}+1} = \theta \frac{1+n}{r_2-n} \cdot \frac{1+r_2}{\lambda(1+\theta)(1+n)+\theta} - \frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r_2)s_{\tilde{t}} \quad \text{(A-33)}$$

$$s_{\tilde{t}} = \frac{\theta}{1+\theta}\left(1 - \frac{p_{\tilde{t}}}{1+n}\right) - \frac{1}{1+\theta} \cdot \frac{p_{\tilde{t}+1}}{1+r_2} \quad \text{(A-34)}$$

Straightforward optimization shows that

$$p_{\tilde{t}} = \theta \frac{1+n}{r_2-n} \cdot \frac{1+r_2}{\lambda(1+\theta)(1+n)+\theta} - \frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r_2)s_{\tilde{t}-1} \quad \text{(A-35)}$$

In period $\tilde{t}-1$, policy is chosen assuming that r is equal to r_1 in period $\tilde{t}$. Hence, the optimization problem is given by:

$$\max_{p_{\tilde{t}-1}} \theta ln[(1+r_1)s_{\tilde{t}-2}+p_{\tilde{t}-1}]+$$

$$+\lambda(1+n)\left\{ln[1-s_{\tilde{t}-1}-\frac{p_{\tilde{t}-1}}{1+n}]+\theta ln[(1+r_1)s_{\tilde{t}-1}+p_{\tilde{t}}]\right\} \tag{A-36}$$

subject to

$$p_{\tilde{t}}=\theta\frac{1+n}{r_1-n}\cdot\frac{1+r_1}{\lambda_2(1+\theta)(1+n)+\theta}-\frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r_1)s_{\tilde{t}-1} \tag{A-37}$$

$$s_{\tilde{t}-1}=\frac{\theta}{1+\theta}\left(1-\frac{p_{\tilde{t}-1}}{1+n}\right)-\frac{1}{1+\theta}\cdot\frac{p_{\tilde{t}}}{1+r_1} \tag{A-38}$$

Straightforward optimization shows that

$$p_{\tilde{t}-1}=\theta\frac{1+n}{r_1-n}\cdot\frac{1+r_1}{\lambda(1+\theta)(1+n)+\theta}-\frac{\lambda(1+\theta)(1+n)}{\lambda(1+\theta)(1+n)+\theta}(1+r_1)s_{\tilde{t}-2} \tag{A-39}$$

The initial jump in the pension benefit follows from comparison of eq. (A-35) with the level of $p_{\tilde{t}}$ if there was no change in r. Straightforward computation shows that $p_{\tilde{t}}$(no change) $>$ $p_{\tilde{t}}$(with change). In a similar way it follows that $s_{\tilde{t}}$(no change) $<$ $s_{\tilde{t}}$(with change).

The effect of a decrease in the growth rate of the population

Unanticipated

The optimization problem in period $\tilde{t}$ is given by

$$\max_{p_{\tilde{t}}} \theta ln[(1+r)s_{\tilde{t}-1}+p_{\tilde{t}}]+$$

$$+\lambda(1+n_2)\left\{ln[1-s_{\tilde{t}}-\frac{p_{\tilde{t}}}{1+n_1}]+\theta ln[(1+r)s_{\tilde{t}}+p_{\tilde{t}+1}]\right\} \tag{A-40}$$

subject to

$$p_{\tilde{t}+1}=\theta\frac{1+n}{r-n_2}\cdot\frac{1+r}{\lambda(1+\theta)(1+n_2)+\theta}-\frac{\lambda(1+\theta)(1+n_2)}{\lambda(1+\theta)(1+n_2)+\theta}(1+r)s_{\tilde{t}} \tag{A-41}$$

$$s_{\tilde{t}} = \frac{\theta}{1+\theta}\left(1 - \frac{p_{\tilde{t}}}{1+n}\right) - \frac{1}{1+\theta} \cdot \frac{p_{\tilde{t}+1}}{1+r} \tag{A-42}$$

Straightforward optimization shows that

$$p_{\tilde{t}} = \theta\frac{1+n_1}{r-n_2} \cdot \frac{1+r}{\lambda(1+\theta)(1+n_2)+\theta} - \frac{\lambda(1+\theta)(1+n_2)}{\lambda(1+\theta)(1+n_2)+\theta}(1+r)s_{\tilde{t}-1} \tag{A-43}$$

In period $\tilde{t}-1$, policy is chosen assuming that n is equal to n_1 in period $\tilde{t}$. Hence, the optimization problem is given by:

$$\max_{p_{\tilde{t}-1}} \theta ln[(1+r)s_{\tilde{t}-2} + p_{\tilde{t}-1}] +$$

$$+ \lambda(1+n_1)\left\{ln[1 - s_{\tilde{t}-1} - \frac{p_{\tilde{t}-1}}{1+n_1}] + \theta ln[(1+r)s_{\tilde{t}-1} + p_{\tilde{t}}]\right\} \tag{A-44}$$

subject to

$$p_{\tilde{t}} = \theta\frac{1+n_1}{r-n_1} \cdot \frac{1+r}{\lambda(1+\theta)(1+n_1)+\theta} - \frac{\lambda(1+\theta)(1+n_1)}{\lambda(1+\theta)(1+n_1)+\theta}(1+r)s_{\tilde{t}-1} \tag{A-45}$$

$$s_{\tilde{t}-1} = \frac{\theta}{1+\theta}\left(1 - \frac{p_{\tilde{t}-1}}{1+n_1}\right) - \frac{1}{1+\theta} \cdot \frac{p_{\tilde{t}}}{1+r} \tag{A-46}$$

Straightforward optimization shows that

$$p_{\tilde{t}-1} = \theta\frac{1+n_1}{r-n_1} \cdot \frac{1+r}{\lambda(1+\theta)(1+n_1)+\theta} - \frac{\lambda(1+\theta)(1+n_1)}{\lambda(1+\theta)(1+n_1)+\theta}(1+r)s_{\tilde{t}-2} \tag{A-47}$$

The initial jump in the pension benefit follows from comparison of eq. (A-43) with the level of $p_{\tilde{t}}$ if there was no change in n. Straightforward computation shows that $p_{\tilde{t}}(\text{no change}) < (>)\ p_{\tilde{t}}(\text{with change})$ iff $s_{\tilde{t}-1} > (<) - \frac{1+n_1}{r-n_1}\left(1 - \frac{\lambda(1+\theta)(1+n_1)+\theta}{\lambda(1+\theta)(r-n_2)}\right)$. In a similar way it follows that $s_{\tilde{t}}(\text{no change}) > (<)\ s_{\tilde{t}}(\text{with change})$ iff $s_{\tilde{t}-1} > (<) - \frac{1+n_1}{r-n_1}\left(1 - \frac{\lambda(1+n_1)+1}{\lambda(r-n_2)}\right)$.

Anticipated

In case of an anticipated change in the population growth rate, the optimal pension benefit in period $\tilde{t}$ is identical to eq. (A-43). The optimization problem in period $\tilde{t}-1$ is, because of the anticipation, given by:

$$\max_{p_{\bar{i}-1}} \theta ln[(1+r)s_{\bar{i}-2} + p_{\bar{i}-1}] +$$

$$+ \lambda(1+n_1)\left\{ln[1 - s_{\bar{i}-1} - \frac{p_{\bar{i}-1}}{1+n_1}] + \theta ln[(1+r)s_{\bar{i}-1} + p_{\bar{i}}]\right\} \tag{A-48}$$

subject to

$$p_{\bar{i}} = \theta\frac{1+n_1}{r-n_2} \cdot \frac{1+r}{\lambda(1+\theta)(1+n_2)+\theta} - \frac{\lambda(1+\theta)(1+n_2)}{\lambda(1+\theta)(1+n_2)+\theta}(1+r)s_{\bar{i}-1} \tag{A-49}$$

$$s_{\bar{i}-1} = \frac{\theta}{1+\theta}\left(1 - \frac{p_{\bar{i}-1}}{1+n_1}\right) - \frac{1}{1+\theta} \cdot \frac{p_{\bar{i}}}{1+r} \tag{A-50}$$

Straightforward optimization shows that

$$p_{\bar{i}-1} = \theta\frac{r-n_2+1+n_1}{r-n_2} \cdot \frac{1+r}{\lambda(1+\theta)(1+n_1)+\theta} -$$

$$- \frac{\lambda(1+\theta)(1+n_1)}{\lambda(1+\theta)(1+n_1)+\theta}(1+r)s_{\bar{i}-2} \tag{A-51}$$

Doing this repeatedly gives for anticipation j periods before:

$$p_{\bar{i}-j} = \theta\frac{1+n_1}{r-n_2} \cdot \frac{1+r+\frac{1+r}{r-n_1}\left[1-\left(\frac{1+n_1}{1+r}\right)^{j-1}\right](n_1-n_2)}{\lambda(1+\theta)(1+n_1)+\theta} -$$

$$- \frac{\lambda(1+\theta)(1+n_1)}{\lambda(1+\theta)(1+n_1)+\theta}(1+r)s_{\bar{i}-j-1} \tag{A-52}$$

The initial jump of the pension benefit at the period the future change in n becomes known follows from comparison of eq. (A-52) with the level of $p_{\bar{i}-j}$ if there was no change. Straightforward computation shows that $p_{\bar{i}-j}$(no change) $>$ $p_{\bar{i}-j}$(with change). In a similar way it follows that $s_{\bar{i}-j}$(no change) $<$ $s_{\bar{i}-j}$(with change).

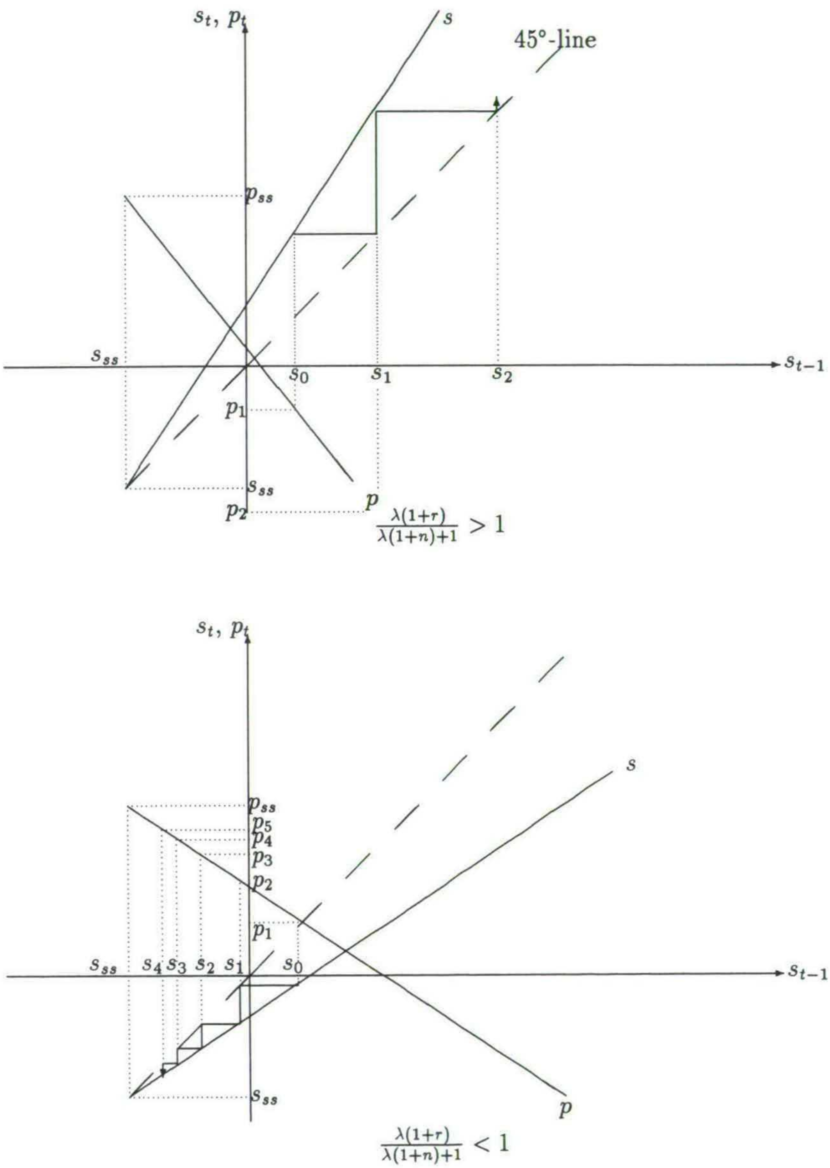


Figure 4.1: The evolution of taxes and transfers

Chapter 5

Public investment policy as an intergenerational conflict

5.1 Introduction

Until the mid eighties public investment and public capital received little attention by economists. Then, two developments put investment in public capital again at the forefront of economic analysis. First, a number of empirical studies appeared, dealing with the question whether the decline in US productivity could be attributed to a shortage in public capital (*cf.* Gramlich (1994) for a critical overview of these studies). Second, in theoretical research, public investment regained interest by the literature on endogenous growth. Investment in public capital, be it infrastructure or education, could raise the productivity of private capital and, hence, raise output, consumption etcetera. It might even affect growth rates permanently (*cf.* Barro and Sala-i-Martin (1992)). Both these developments led to a boost in empirical and theoretical research on the role of public capital. Besides, policymakers are becoming increasingly aware of the role public capital can play in the competiveness of an economy. Especially in Europe where the increased economic integration leads to more and more harmonization of fiscal policies. This makes this traditional instrument of competition between countries, *e.g.* by offering favourable tax treatments to foreign investors, less useful. Competition between countries shifts to the provision of the best environment for firms to operate. Infrastructure is an important aspect of this.

If public capital is important for productivity, growth and competition, the question what determines the level of investment in public capital becomes important as well. Yet, this question is hardly addressed. In most of the existing literature dealing with public capital, either the utility of a representative agent or a social welfare function consisting of utilities of all present and future generations is optimized. There are a few exceptions which deal with some form of political decision making on public investment. Alesina and Rodrik (1994) develop a model where infinitely-lived individuals differ with respect to their relative share of the capital stock. The government uses a tax on capital to finance investment in public capital which enters the private production function (similar to Barro (1990)). The level of public capital is decided upon by majority voting. Thus, the level of public capital is the result of a political conflict between individuals who differ in their share of the capital stock. The level chosen is suboptimal in the sense that it falls below the level that maximizes growth. The growth maximizing level would be preferred by a 'pure' capitalist, *i.e.* an individual that only has capital income and no labour income. If the median voter has more labour income, he prefers a higher tax on capital. Furthermore, the more unequal the distribution of income and wealth, the lower the rate of growth. In Van der Ploeg and Van de Klundert (1991), short-sighted politicians, who have a higher rate of time preference than the private sector, increase public consumption at the expense of public investment. This gives lower economic growth. Another study dealing with investment in public capital in a politico-economic framework is Konrad (1993). He models the political process as a gerontocracy, *i.e.* only the current elderly have political power. They decide on a social security tax and on investment in public capital. There is investment in public capital because it is productive immediately and therefore raises the tax base of the social security tax. Another explanatory factor for investment in public capital may be altruism. *E.g.* in Glomm and Ravikumar (1992) current generations leave a bequest to their offspring because of altruistic feelings. The bequest is the quality of (public or private) schools. This could also be applied to public capital. In Jappelli and Ripa-di-Meana (1994), the level of public capital is determined in a representative democracy by a government that only cares about the current generations. Public capital, which is productive immediately, only benefits the current young generation. Besides public capital, there is also public consumption which benefits both generations. Both these public expenditures are financed by public debt and a lump-sum tax levied on the current young generation only.

Their conclusion is that the level of public capital declines if more weight is attached to the utility of the current elderly[1].

What determines the level of investment in public capital is also the focus of this chapter. Besides, whether there will be under- or overinvestment relative to a bench-mark level is analysed as well. In this chapter the level of investment is determined in a conflict between young and old generations. The government operates in a representative democracy setting where only the present living generations are involved in the decision making on public investment. Since for most public investment projects it takes considerable time before they are ready for use, older generations do benefit only partly or maybe even not al all from public capital and therefore will be opposed to investment in public capital. Due to investment in public capital, the interest revenues on old-age savings of the current young generations may increase. Besides, there may be spillovers to future generations in the form of higher wages. This increases the future tax base making possible, *e.g.* higher pensions. This may also be beneficial for the current young. The opposition of the elderly and the non-representation of future generations, who may benefit from public investment as well because of an increase in wages, may, therefore, lead to underinvestment. However, this result is closely connected to the way in which these investments are financed. If the budget has to be balanced and only current generations pay taxes, underinvestment is likely to be the case. If, however, the government can use public debt to finance public investment, this may lead to overinvestment since, in this case, the costs can be shifted onto the shoulders of future, yet unrepresented, generations.

[1]The study of Jappelli and Ripa-di-Meana (1994) contains some serious flaws. First, in their command solution they optimize welfare of a representative generation. The solution then is determined by the golden rule condition. In their market solution, however, they consider a welfare function of the two living generations, *i.e.* the current young and old generation. In this set-up there are two sources for differences between both solutions. On the one hand, there is the difference in the objective function. In the command solution, the current elderly are neglected. But in the market solution they are taken into account. On the other hand, there is the difference between instruments (command versus market). However, because the elderly do not pay taxes in their model, the government has sufficient instruments to attain the command solution through the market if they neglect the elderly completely. Hence, their paper compares two governments with different objectives. The difference between command and market solution is superfluous. Second, they solve their model implicitly restricting themselves to a set of solutions where the policy is constant over time. This solution need not to coincide with the steady state of the unrestricted model as they erroneously suggest.

The resulting level of public capital due to the conflict between the generations present is analyzed in Section 5.4. Three cases are distinguished. One where the government has to balance its budget and can use time-dependent taxes only to finance public capital. In the second case, the government still has to balance its budget but has the ability to tax both generations differently. This implies that the government can affect the welfare distribution between the two generations not only by public capital but also by taxing both generations at a different level. The final case is where the government is allowed to use debt to finance public capital. Before that, in Section 5.2, an overlapping generations model of a closed economy is developed. The private sector consists of two overlapping generations of which only the young generation works. The production side of the economy consists of an infinite number of price-taking firms. Public capital enters the private production function. The government is modeled in a representative democracy setting and maximizes utility of present generations only. Section 5.3 discusses the problems of defining a command solution and presents a command optimum for this representative government which is used as a bench-mark in the following sections. Section 5.5 concludes this chapter.

5.2 The economy

5.2.1 The private sector

The private sector consists of overlapping generations of homogeneous agents. Each agent lives for two periods, the first period, when young, labeled 'y', the second period, when old, labeled 'o'. There is no population growth and the size of each generation is normalized and equal to one. Each agent optimizes an additive separable lifetime utility function of consumption in both periods. First-period consumption (c_t^y) of an individual born at time t equals wages (w_t) earned minus a lump-sum tax (τ_t^y) and minus savings (s_t). Second-period consumption (c_{t+1}^o) equals the first-period savings plus interest revenues minus a lump-sum tax (τ_{t+1}^o). Agents choose a level of savings that optimizes lifetime utility. This gives the following optimization problem:

$$\max U_t = u_t(c_t^y) + \theta v_t(c_{t+1}^o) \tag{5.2.1}$$

$$s.t.\ c_t^y = w_t - \tau_t^y - s_t \tag{5.2.2}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t - \tau_{t+1}^o \tag{5.2.3}$$

where U_t denotes the lifetime utility function of a generation born at time t. u_t and v_t are the instantaneous utility functions of the first and second period of their lifes respectively. θ is the private discount factor and r_{t+1} is the interest rate in period $t+1$. Straightforward optimization shows that optimal savings s^* are determined by:

$$\frac{\partial u_t}{\partial c_t^y} = \theta(1 + r_{t+1})\frac{\partial v_t}{\partial c_{t+1}^o} \tag{5.2.4}$$

Thus, given the interest rate and the discount factor, consumers equalize the present values of marginal utilities. In the sequel, the utility function is assumed to be logarithmic in consumption in both periods, i.e $u_t = lnc_t^y$ and $v_t = lnc_{t+1}^o$. This implies that savings are given by:

$$s_t = \frac{\tau_{t+1}^o}{(1+\theta)(1+r_{t+1})} + \frac{\theta}{1+\theta}(w_t - \tau_t^y) \tag{5.2.5}$$

Immediately it follows that savings decrease if the interest rate or the tax paid when young, τ^y, rise. Savings increase if wages or old-age taxes rise. In this model, savings can be interpreted as pension contributions into a capital reserve system. When old, the individuals receive a pension which equals the contribution plus the interest revenues.

5.2.2 Firms

The production side of the economy is assumed to consist of a large number of price-taking firms, each optimizing profits. There are three inputs for production, labour (l), private capital (k) and public capital (g). Capital (private and public) is assumed to depreciate fully within one period. Thus, the flow of investments in period t equals the stock of capital in period $t+1$. This gives:

$$\max \quad y_t - r_t k_t - w_t l_t - k_t \tag{5.2.6}$$

$$s.t. \quad y_t = f(l_t, k_t, g_t) \tag{5.2.7}$$

where $f(.)$ is a constant returns to scale production function. $f(.)$ satisfies the standard Inada conditions[2] and has declining marginal productivity in both types of capital. Each

[2] $f(\vec{0}) = 0$, $\lim_{x \to 0} f'(\vec{x}) \to \infty$, $\lim_{x \to \infty} f'(\vec{x}) = 0$.

type of capital raises the marginal productivity of the other type, *i.e.* $\frac{\partial y}{\partial k \partial g}$ is positive. The firms take the level of public capital as given. Assuming perfect competition in the factor markets implies that capital is paid according to its marginal productivity. Thus:

$$1 + r_t = \frac{\partial y_t}{\partial k_t} \tag{5.2.8}$$

Because of the externality in the form of public capital, firms make profits. The zero profit condition implies that these profits are included in the wages, *i.e.* given to the current young[3]. Thus:

$$w_t = l_t \frac{\partial y_t}{\partial l_t} + g_t \frac{\partial y_t}{\partial g_t} \tag{5.2.9}$$

The second term on the right-hand-side of eq. (5.2.9) are the profits due to the free availability of public capital. The production function is assumed to be Cobb-Douglas and given by:

$$y_t = A l_t^{\gamma} k_t^{\alpha} g_t^{\beta} \tag{5.2.10}$$

where $\gamma > 0$ is the elasticity of labour, $\alpha > 0$ and $\beta > 0$ are the elasticities of private and public capital respectively and A is a scale parameter. Since $\gamma + \alpha + \beta = 1$ it holds that $\alpha + \beta < 1$ implying that growth is not endogenous. Furthermore, the supply of labour is assumed to be inelastic. Hence, labour can be normalized to one. Eq. (5.2.10) implies that

$$1 + r_t = \frac{\alpha y_t}{k_t} \tag{5.2.11}$$

The profit due to the externality is equal to βy. Since this is included in the wages, wages equal

$$w_t = \gamma y_t + \beta y_t = (1 - \alpha) y_t \tag{5.2.12}$$

Note that g is not a pure public good. Each firm gets its private share of public capital. If it were a pure public good, each firm could increase its profit by splitting up in smaller

[3]If profits are positive or negative, firms would either produce at an infinite scale, attaining infinite profits, or contract its scale to 0 (*cf.* Barro and Sala-i-Martin (1995), p.68). Since labour supply is exogenous, including these profits into the wages creates no additional distortionary effects.

production units. By assuming g to be a rival good, this is avoided.

5.2.3 Government

The level of public capital is chosen by the government. The government is modelled in a representative democracy setting. It optimizes:

$$U_{t-1}(c^y_{t-1},\ c^o_t) + \lambda U_t(c^y_t,\ c^o_{t+1}) \tag{5.2.13}$$

i.e. the government takes account of present generations only. λ denotes the relative political weight of the young generation[4]. In this set-up, each period, a new government optimizes utility of the then living generations. Since future policy choices affect the utility of the current young, behavioural assumptions with respect to future policy choices are necessary. It will be assumed that current governments conjecture that future policy instruments, *i.e.* future tax or debt levels are not affected by current policy instruments. Hence, Nash behaviour is assumed. However, all other effects are taken into account. With respect to the private sector and the capital market, the government acts as a Stackelberg leader. It thus takes the behaviour of the private sector as reflected by eqs. (5.2.4) and (5.2.8) into account.

The government faces a budget identity, which in its most general form is given by:

$$g_{t+1} + (1 + r_t)b_{t-1} = \tau^o_t + \tau^y_t + b_t \tag{5.2.14}$$

where b_t is the stock of government debt at the end of period t. Eq. (5.2.14) encompasses the various sets of policy instruments discussed in the sequel. From this budget constraint follows that it is assumed that public capital becomes productive with a lag of one period.

5.2.4 Closing the model

Finally, equilibrium on the capital market requires:

$$s_t = k_{t+1} + b_t \tag{5.2.15}$$

[4]In previous chapters, λ denoted the relative political weight of a young individual. However, since population growth is absent in this chapter, this equals the relative political weight of the young generation.

Hence, private capital, just like public capital, becomes productive with a lag of one period.

5.2.5 The effects of public capital

Before continuing with a bench-mark situation and the market solutions, a closer look at the effects of public capital in this model is in order. In a Neoclassical model like this, investment in public capital has several opposing effects. These effects can be divided into financial effects and real effects[5]. The financial effects relate to the way in which the investment expenditures are financed, by debt or taxation. From the budget constraint of the government, eq. (5.2.14), follows that, in this model, an increase in g can be financed either by an increase in taxation of the young (τ^y), an increase in taxation of the elderly (τ^o) or an increase in public debt (b). Focusing on each of these instruments in isolation, then, first, an increase in g, financed completely by an increase in τ^y leads, ceteris paribus, to a decrease in private savings. This follows immediately from eq. (5.2.5). From the equilibrium condition on the capital market, eq. (5.2.15), follows that this leads to an decrease in the supply of private capital. This decrease in k has, ceteris paribus, two effects. On the one hand, it increases the interest rate, since the marginal product of capital is declining in k. This gives a decline in savings. On the other hand, a lower k decreases wages w (see eq. (5.2.12)) because labour supply is inelastic. Hence, a decline in wages decreases savings as well (all ceteris paribus). Figure 5.1 summarizes. The overall effect on private savings is negative. Moreover, savings again affects private

$$\begin{aligned} \tau^y \uparrow \rightarrow s \downarrow \rightarrow k \downarrow & \rightarrow r \uparrow \rightarrow s \downarrow \\ & \rightarrow w \downarrow \rightarrow s \downarrow \end{aligned}$$

Figure 5.1: The effects of g financed by τ^y.

capital. As a result, multiplier effects may occur. Financing the extra amount of public capital completely by an increase in public debt b gives similar effects because an increase in the level of public debt leads, ceteris paribus, to a decrease of private capital since the capital market is in equilibrium. Using τ^o to finance the extra amount of g gives exactly opposite effects. From eq. (5.2.5) follows that an increase in τ^o leads, ceteris paribus,

[5]This labelling is taken from Aschauer (1988), dealing with a more general discussion of the effects of fiscal policy in a Neoclassical model.

to an increase in private savings. The rest immediately follows from figure 5.1, with the opposite signs. In general, it is more likely that governments use a combination of these instruments. This implies that the different effects have to be weighed against each other.

So far, only the financial effects have been discussed. There may also be real effects. More public capital increases wages and interest rates, which has an ambiguous effect on savings by the consumers. Besides, there are crowding-in and crowding-out effects on the firm level. On the one hand, depending on the degree of complementarity between private and public capital in production, public capital may raise the productivity of private capital. *E.g.* better roads raise the productivity of trucks using them. Thus, more public capital leads to a crowding-in of private capital. On the other hand, depending on the degree of substitutability between private and public capital, an increase in the level of public capital may raise the overall level of capital above its desired level by the firms who, ceteris paribus, will respond with a decrease of private capital. Thus, public capital crowds-out private capital.

Thus, the effect of public capital on the economy is ambiguous due to the several, financial and real, effects. When analysing investment in public capital, these effects have to be weighed against each other. Besides, the effect on wages is an externality since this benefits future generations for which current governments do not care. Finally note that the analysis is confined to steady states. This implies that effects like the spillover to future generations have to be taken into account when analysing steady states.

5.3 A command solution

An important and often heard question is whether there is over- or underinvestment in public capital. In order to compare the different sets of policy instruments used in the following section and to analyse which set gives solutions closest an 'optimal' solution, a bench-mark case has to be defined. This is a so-called command solution, where the government, as a social dictator, can directly choose consumption and capital levels. The 'a' in the title of this section has been chosen deliberately. This is not a command solution in the usual sense. Usually, a command solution in an overlapping generations model implies an infinitely long living social dictator optimizing a social welfare function including utilities of all present and future generations where the dictator simultaneously chooses all present and future consumption levels. The steady-state values in that case follow from the modified golden rule condition. However, in the present context of a representative democracy, there is a sequence of governments, each living for one period and optimizing a decision function containing only the utilities of the generations living in that period and not the utilities of future generations. Besides, in a social welfare function, utilities of future generations are discounted by a social discount factor, whereas in the representative democracy, the current young are weighed with their relative political power and future generations are not involved and have therefore no weight at all. The social discount factor is usually restricted to be between 0 and 1, the political weight can, in principle, take any (positive) value. Thus, comparing the outcome of a traditional command solution to the market solution implies not only analysing whether a given set of policy instruments suffices to replicate the command solution through the market but, also, comparing two different objective functions. To avoid differences caused by different objective functions, one should derive a command solution that follows from maximizing eq. (5.2.13). That is, one should maintain the assumption that the government is not interested in the utility of future generations and, at the same, assume that it is able to choose current consumption levels. However, then complications arise because what to do with c^o_{t+1}? Since the scope for such a myopic dictator in period t is confined to period t only, c^o_{t+1} is chosen by the myopic dictator in the following period (as c^o_t is chosen by the present myopic dictator). But, c^o_{t+1} gives utility to the current young generation, a generation the present myopic dictator cares about. Hence, behavioural assumptions with respect to c^o_{t+1} have to be made. As seen in Chapter 2, this can either be Nash, taking c^o_{t+1} as given, or Stackelberg, taking the behavioural response of the following myopic dictator into account. Both assumptions and its implications for steady-state values will be analysed. But, first, the steady state for a dictator optimizing an infinite horizon social welfare function is given. This is the more standard command solution and it will be compared to the solution chosen by the myopic dictator.

Social welfare. When a dictator optimizes social welfare, it maximizes

$$\max \sum_{t=0}^{\infty} \rho^t[\theta v(c^o_t) + \rho u(c^y_t)] \tag{5.3.1}$$

where $0 < \rho < 1$ is the social discount factor. Each period the social dictator has to obey the resource constraint, $y_t = c_t^o + c_t^y + k_{t+1} + g_{t+1}$. This resource constraint can be used to substitute out c_t^o. Then, optimization with respect to c_t^y, k_{t+1}, g_{t+1} gives the following first-order conditions:

$$\theta \frac{dv_{t-1}}{dc_t^o} = \rho \frac{du_t}{dc_t^y} = \rho\theta \frac{dv_t}{dc_{t+1}^o} \cdot \frac{\partial y_{t+1}}{\partial k_{t+1}} = \rho\theta \frac{dv_t}{dc_{t+1}^o} \cdot \frac{\partial y_{t+1}}{\partial g_{t+1}} \qquad t = 1, \ldots, \infty \tag{5.3.2}$$

The first equality denotes the optimal intergenerational distribution of welfare between current elderly and young. The second equality denotes the optimal intertemporal allocation of welfare of an individual. The last equality implies an efficiency condition: The levels of private and public capital[7] are chosen such that their marginal products are equal. For logarithmic utility and a Cobb-Douglas production technology the steady state is given by

$$\begin{aligned} g &= \rho\beta y & c^o &= \tfrac{\theta}{\rho+\theta}[1 - \rho(\alpha+\beta)]y \\ k &= \rho\alpha y & c^y &= \tfrac{\rho}{\rho+\theta}[1 - \rho(\alpha+\beta)]y \\ y &= \left[A(\rho\alpha)^\alpha(\rho\beta)^\beta\right]^{\frac{1}{1-\alpha-\beta}} \end{aligned} \tag{5.3.3}$$

This steady state follows from the modified golden rule condition, i.e. $\frac{\partial y}{\partial k} = \frac{\partial y}{\partial g} = \frac{1}{\rho}$. The efficiency condition implies that the ratio of k over g equals ${}^{\beta}/_{\alpha}$.

Nash. A sequence of myopic dictators operating in a representative democracy setting, optimizes

$$\max_{c_t^y, k_{t+1}, g_{t+1}} \theta v(y_t - c_t^y - k_{t+1} - g_{t+1}) + \lambda[u(c_t^y) + \theta v(c_{t+1}^o)] \tag{5.3.4}$$

Nash behaviour implies taking c_{t+1}^o as given. As a result, there is no investment. Capital becomes productive only the following period. But since c_{t+1}^o is taken as given, this implies that there is no investment in capital. Moreover, the myopic dictator of the

[7]Note that the labels 'private' or 'public' are, of course, not relevant here. For the social dictator it are just two types of capital, both necessary for production.

previous period neither chooses a positive amount of capital, there is no production and, thus, no consumption. Hence, there is no internal steady state in this case[8].

Stackelberg. In case of Stackelberg behaviour, the reaction of the next period myopic dictator on current choices is fully taken into account. By using the resource constraint, the optimal choice of c^o_{t+1} by the next myopic dictator can be written as a function of y_{t+1}: $c^o[y_{t+1}]$. Hence, the current myopic dictator optimizes

$$\max_{c^y_t, k_{t+1}, g_{t+1}} \theta v(y_t - c^y_t - k_{t+1} - g_{t+1}) + \lambda[u(c^y_t) + \theta v(c^o[y_{t+1}])] \tag{5.3.5}$$

The first-order conditions are given by

$$\theta \frac{dv_{t-1}}{dc^o_t} = \lambda \frac{du_t}{dc^y_t} = \lambda\theta \frac{dv_t}{dc^o_{t+1}} \cdot \frac{dc^o_{t+1}}{dy_{t+1}} \cdot \frac{\partial y_{t+1}}{\partial k_{t+1}} = \lambda\theta \frac{dv_t}{dc^o_{t+1}} \cdot \frac{dc^o_{t+1}}{dy_{t+1}} \cdot \frac{\partial y_{t+1}}{\partial g_{t+1}} \tag{5.3.6}$$

Note that the efficiency condition holds here as well. Compared to the social welfare maximizing dictator, the relation between the marginal utility of consumption of the current elderly ($\frac{\partial v_{t-1}}{\partial c^o_t}$) and the marginal utility of first-period consumption of the current young ($\frac{\partial u_t}{\partial c^y_t}$) is identical if $\lambda = \rho$. The difference is in the optimal intertemporal allocation of utility of an individual. Because of the resource constraint, for a social welfare maximizing dictator it holds that $c^o_{t+1} = y_{t+1} - c^y_{t+1} - k_{t+2} - g_{t+2}$. Since for the social welfare optimizing dictator, c^y_{t+1}, k_{t+2} and g_{t+2} are instruments as well for optimizing eq. (5.3.1), it holds that $\frac{dc^o_{t+1}}{dy_{t+1}} = 1$. This implies that a social welfare maximizing dictator acts as if an extra unit of production is completely given to the next-period elderly, *i.e.* the current young when old. However, now, c^y_{t+1}, k_{t+2} and g_{t+2} are instruments of the next-period myopic dictator. It is by no means sure that an extra unit of production is completely given to the elderly of that period. Hence, investment generates an externality in the form of a spillover to future generations because they benefit from the extra production as well. Thus, $\frac{dc^o_{t+1}}{dy_{t+1}}$ does not have to be equal to one.

For logarithmic utility and a Cobb-Douglas production technology, the steady state is determined by:

[8]This result is driven by the fact that with a Cobb-Douglas production technology, g is a necessary input.

$$(a)\ c^y = \frac{\lambda}{\theta}c^o, \quad (b)\ k = \frac{\alpha}{\beta}g$$

$$(c)\ \frac{dc^o}{dy} = \frac{1}{\lambda\alpha}\frac{k}{y} \quad (\frac{dc^o}{dy} = \frac{1}{\lambda\beta}\frac{g}{y})$$

Eq. (a) follows from the first equality of eq. (5.3.6), eq. (b) from the third and eq. (c) from the second. The term $\frac{dc^o}{dy}$ is problematic, since its exact specification is unknown. By using the method of undetermined coefficients it is possible to solve for the steady state. Assume $\frac{dc^o}{dy}$ is constant in steady state and equal to δ. Then the steady state is given by

$$g = \lambda\beta\delta y \qquad c^o = \tfrac{\theta}{\lambda+\theta}[1-\lambda\delta(\alpha+\beta)]y$$

$$k = \lambda\alpha\delta y \qquad c^y = \tfrac{\lambda}{\lambda+\theta}[1-\lambda\delta(\alpha+\beta)]y \tag{5.3.7}$$

$$y = \left[A(\lambda\alpha\delta)^\alpha(\lambda\beta\delta)^\beta\right]^{\frac{1}{1-\alpha-\beta}}$$

δ follows from $\frac{\partial c^o}{\partial y} = \frac{\theta}{\lambda+\theta}[1-\lambda\delta(\alpha+\beta)] = \delta$ and it is given by $\delta = \frac{\lambda}{\lambda+\theta+\lambda^2(\alpha+\beta)}$. Note that $\delta < 1$. This implies that an extra unit of production is only partially given to the next-period elderly, *i.e.* the current young when old. The rest is divided over consumption of the next-period young and private and public capital. Note that the efficiency condition requires that $\frac{g}{k} = \frac{\beta}{\alpha}$. From the steady-state values, eqs. (5.3.3) and (5.3.7), follows that if $\rho = \lambda$, the level of g, k and y is smaller than with a social welfare maximizer. Since the production technology exhibits decreasing returns to scale with respect to k and g, c^y and c^o will be lower as well. If $\lambda\delta = \rho$, the steady-state levels for g, k and y are the same. However, c^y will be higher and c^o will be lower with Stackelberg behaviour. Though total consumption ($c^y + c^o$) with Stackelberg behaviour equals total consumption with the social welfare maximizer, the division over the generations differs. With Stackelberg behaviour it is divided according to $\frac{c^o}{c^y} = \frac{\theta}{\lambda}$, with a social welfare maximizer according to $\frac{c^o}{c^y} = \frac{\theta}{\rho}$ which equals $\frac{\theta}{\lambda\delta}$ in this case. In order to obtain the same capital and production levels as with a social welfare maximizing dictator, the myopic dictator has to give a much larger weight to the current young. This larger weight 'compensates' for the disregarding of future generations and implies a higher c^y relative to c^o.

What does this imply for the bench-mark case to be used in the sequel of this chapter? As noted in the introduction, Jappelli and Ripa di Meana (1994) choose for their command solution the optimization of utilities of a representative generation, resulting in steady-state levels determined by the golden rule condition. However, a justification for this choice is absent. Besides, the fact that the current elderly are fully neglected makes this solution unsatisfactory as a bench-mark case. What then? Which bench-mark case is to be used should, of course, be closely connected to the choice of the behavioural assumptions used with respect to future policy choices. In this chapter it is assumed that current governments take future policy choices as given, *i.e.* Nash behaviour. Thus, the effects of current policy choices on future policy choices is not taken into account. However, as seen above, the command solution in a representative democracy with Nash behaviour with respect to future policies is not defined. Therefore, as a bench-mark for the sequel of this chapter the Stackelberg case will be taken. Though this implies that differences between the command and market solutions can also stem from the difference in behavioural assumptions (Stackelberg in the command solution and Nash in the market solution), it is the closest related. The objective functions for this command solution and the market solution are identical. Besides, the effect of current policy choices on future consumption of the present young is taken into account in both cases. Table 5.1 provides values for the baseline case[9]. The efficiency condition implies $\frac{g}{k} = 1$ for the

g	k	y	g/k	U	c^y	c^o
1.043	1.043	10.170	1.000	2.231	5.215	2.869

Table 5.1: Baseline in case of Stackelberg behaviour.

baseline case. Table 5.2 provides the effects of changes in λ for the baseline case. Thus, an increase in λ leads to more public and private capital. Because of the increased political weight of the current young, the future, *i.e.* the old-age utility of the current young becomes more important. This explains the increase in private and public capital. The efficiency condition is independent of λ and is therefore unaffected.

[9]This and future baseline cases are determined by $\alpha = \beta = 0.2$, $\theta = 0.55$ and $\lambda = 1$. The latter value implies equal weight for both generations. Estimates of the productivity of private capital α are around 0.2. Estimates of the productivity of public capital vary between 0 and 0.4. More recent estimates suggest that 0.2 is more plausible (*cf.* Gramlich (1994)). A θ of 0.55 for a generation (*i.e.* about 30 years) implies an annual discount factor of about 0.98.

	g	k	y	g/k	U	c^y	c^o
$\lambda = 0.8$	0.685	0.685	8.596	1.000	2.048	4.282	2.944
1.0	1.043	1.043	10.170	1.000	2.231	5.215	2.869
1.2	1.428	1.428	11.531	1.000	2.335	5.949	2.727

Table 5.2: The effects of variations in λ.

5.4 The decentralized optimum

In reality, a government cannot directly set consumption and capital levels. Instead, it is restricted by the available policy instruments. Then, what are the implications of different sets of policy instruments for the level of public capital? Will there be over- or underinvestment in public capital, both in level and with respect the efficiency condition? Three sets of policy instruments are discussed in the sequel. First, a balanced budget policy with a lump-sum tax which varies over periods but not within periods between generations, *i.e.* it is time-dependent only. Second, a balanced budget policy with a time- and age-dependent lump-sum tax is considered. Finally, debt-financing of public capital is discussed.

But first, it is interesting to look at the values for debt and taxes that replicate the command solution as given in table 5.1. Taking into account the budget constraint of the government, eq. (5.2.14), the behaviour of the consumers and the firms, eqs. (5.2.5), (5.2.11), (5.2.12), and the equilibrium condition for the capital market, the following values for the tax and debt levels in the baseline case can be computed:

$$\tau^y = 1.509 \qquad \tau^o = -0.119 \qquad b = 0.367$$

Thus, in order to obtain the bench-mark solution, public investment is financed by the current young and, because debt is used, by the future young. The elderly receive a (pension) transfer from the young. The (pension) transfer to the elderly occurs because the profits due to public capital are included in the wages which makes these relatively high. This prevents an inefficiently high level of savings. Besides, the current elderly do not benefit from investment in public capital.

However, in a representative democracy setting, allowing a government to use public debt may lead to extremely large levels of debt if the future burden of debt is not taken into account by the current government. Therefore, in the section where debt finance of public investment is allowed, the government is restricted in its use of public debt. Such a regime is the case in many western countries.

5.4.1 A balanced-budget policy with time-dependent taxes

Assume that the government has to balance its budget each period and cannot discriminate between the different generations, *i.e.* $\tau^y = \tau^o = \tau$. Thus, the government optimizes eq. (5.2.13) subject to

$$g_{t+1} = 2\tau_t \tag{5.4.1}$$

Hence, present generations pay half of the costs of the investment each. Thus, investment in public capital redistributes from the current elderly to the current young because, of these two generations, only the young benefit from public investment. Using the budget constraint to eliminate τ_t, optimization with respect to g_{t+1} gives the following first-order condition:

$$\theta \frac{\partial v_{t-1}}{\partial c_t^o}[-1/2] + \lambda \frac{\partial u_t}{\partial c_t^y}[-1/2] + \lambda\theta \frac{\partial v_t}{\partial c_{t+1}^o}\left[s_t \frac{d(1+r_{t+1})}{dg_{t+1}}\right] = 0 \tag{5.4.2}$$

The first two terms denote the marginal costs in terms of utility of an extra unit of public capital. Since the old and the young generation pay half of the costs of the investment each, these marginal costs are split up between these two generations. The first term denotes the marginal costs in terms of utility for the current elderly, the second term for the current young. The third term denotes the marginal benefit in terms of utility of an extra unit of public capital. Of the current generations only the young benefit from public investment by a (positive) income effect[11]. This income effect is caused by a change in the next-period interest rate which is denoted $\frac{d(1+r_{t+1})}{dg_{t+1}}$. Using that eq. (5.2.4) gives savings s_t as a function of the interest rate $(1 + r_{t+1})$ and the tax rate τ_t, and using the equilibrium condition for the capital market ($s_t = k_{t+1}$), gives:

[11] Public investment also gives a substitution effect between current and future consumption ($\frac{\partial u_t}{\partial c_t^y} \cdot \frac{\partial c_t^y}{\partial s_t} \cdot \frac{ds_t}{dg_{t+1}} + \theta \frac{\partial v_t}{\partial c_{t+1}^o} \cdot \frac{\partial c_{t+1}^o}{\partial s_t} \cdot \frac{ds_t}{dg_{t+1}}$). This effect on utility, however, disappears because of the optimality of private savings (this immediately follows from the first-order condition of the private sector).

$$\frac{d(1+r_{t+1})}{dg_{t+1}} = \frac{\frac{\partial^2 y_{t+1}}{\partial k_{t+1}\partial g_{t+1}} + \frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2} \cdot \frac{\partial s_t}{\partial \tau_t} \cdot \frac{\partial \tau_t}{\partial g_{t+1}}}{1 - \frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2} \cdot \frac{\partial s_t}{\partial (1+r_{t+1})}} \tag{5.4.3}$$

Public capital affects the interest rate which in turn affects private savings. Since private savings again affect the interest rate, a multiplier effect appears which is given in the denominator. In case of logarithmic preferences, $1+r$ negatively affects savings (see eq. (5.2.5)) because substitution effect and income effect cancel each other out and, hence, a negative effect on lifetime income remains[12]. Because of this negative effect on savings, the denominator is smaller than 1, giving a positive multiplier effect. The numerator contains the direct effect of public capital on the marginal product of private capital and the indirect effect on the marginal product of private capital because of the effect of taxes on savings. Since it was assumed that public capital raises the productivity of private capital, the direct effect is positive. The indirect effect is positive as well since an increase in public capital and, thus, in current taxes, leads to a decline in savings, ceteris paribus. This, in turn, positively affects the interest rate. Hence, the total effect of public capital on the interest rate is positive.

When discussing the effects of investment in public capital, many opposing effects are present, including multiplier effects. Even for logaritmic utility functions and a Cobb-Douglas production function, no analytical solutions can be derived. The equations determining the steady-state are given in the appendix. For further analysis simulations have to be used[13]. Table 5.3 presents the resulting levels of g, k and y as well as the g/k-ratio, the level of utility derived by a generation (U) and the consumption levels obtained (c^y and c^o, respectively) for the baseline case. The results obtained for the baseline case hold for other parameter values as well. Comparing the simulation results obtained with the bench-mark level of public capital, clearly indicates underinvestment in public capital, both in level and relative to the level of private capital. The latter implies that the efficiency condition is not fulfilled. What are the reasons for these results? First, note that, when choosing a level of public investment, the government cannot

[12] Because lifetime income, $w_t - \tau_t - \frac{\tau_{t+1}}{1+r_{t+1}}$, decreases.

[13] In these and upcoming simulations, various parameters settings are used. The elasticities of output with respect to the two types of capital are chosen between 0.1 and 0.4. The political discount factor was chosen between 0.4 and 1.2. The private discount factor was chosen between 0.4 and 0.7. Taking one period equal to about 30 years this comes down to an annual discount factor between .96 and .99.

	g	k	y	g/k	U	c^y	c^o	
	0.722	3.675	12.156	0.197	2.139	5.688	2.070	
(Benchmark:	1.043	1.043	10.170	1.000	2.231	5.215	2.869	)

Table 5.3: Baseline if $g = 2\tau$.

exempt the current elderly from taxation though they do not benefit from it, since the government is forced to tax both generations at the same level. Therefore, compared to the mix of instruments given at the beginnning of this section, leading to the bench-mark allocation, τ^o is too high, and τ^y is too low. This increases savings. Second, the private sector chooses savings such that the intertemporal allocation of consumption obeys $\frac{c^o}{c^y} = \theta(1+r)$, which follows from the first-order condition, eq. (5.2.4). However, the disregarding of the negative effect on the private interest rate by the private sector leads to 'too much' saving. The high level of savings and the impossibility of exempting the current elderly from paying for public investment, leads to underinvestment in public capital, both in level and relative to the level of private capital. The latter implies that the g/k-ratio is too low and, thus, inefficient.

What are the effects of changes in the political and private preferences on the level of public capital? The effects on the steady-state levels of an increase in the relative political weight of the present young (λ) are reported in table 5.4 for the baseline case. Table 5.5 presents the results where a whole range of values for the exogenous parameters is considered. Since the changes in table 5.4 have the same sign as in table 5.5, it can be concluded that the results are robust for the range of parameter values used. With an

	g	k	y	g/k	c^y	c^o	$1+r$	w
$\lambda = 0.8$	0.569	3.416	11.422	0.167	5.437	2.000	0.669	9.138
1.0	0.722	3.673	12.154	0.197	5.688	2.070	0.661	9.723
1.2	0.873	3.902	12.778	0.224	5.883	2.119	0.655	10.222

Table 5.4: The effects of variations in λ around the baseline case.

increase in λ, the interests of the present young generation get a higher weight. Comparing steady states, this implies redistribution from the current elderly to the current

	g	k	y	$^g/_k$	c^y	c^o	$1+r$	w
λ	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$+$

Table 5.5: The effects of an increase in λ (general).

young. Hence, the level of public capital increases. Though the associated taxes affect both generations, the current young get more public capital in return. In the long run (note that steady states are compared) wages increase due to an increase in g. This causes savings to increase as well. But, since this is a second order effect in response to g, savings and, thus, k rises less than g. Hence, the $^g/_k$-ratio increases. The increase in production leads to higher levels of consumption c^y and c^o but c^y increases relatively more than c^o. Note that this implies that for the steady state increasing the political weight of the current young generation is beneficial for the current elderly as well.

Tables 5.6 and 5.7 report the effects of an increase in the private discount factor. Table 5.6 provides values for g, k, y and the g/k-ratio around the baseline case. Table 5.7 provides more general results, where other values for the other exogenous parameters are taken into account as well. Again, comparing the two tables, it follows that the results are robust. An increase in θ implies in the first place a stronger preference of the private sec-

	g	k	y	$^g/_k$	c^y	c^o	$1+r$	w
$\theta = 0.40$	0.587	2.673	10.941	0.219	5.787	1.895	0.819	7.842
0.55	0.722	3.673	12.154	0.197	5.688	2.070	0.661	9.723
0.70	0.821	4.537	13.008	0.181	5.459	2.191	0.573	10.406

Table 5.6: The effects of variations in θ around the baseline case.

	g	k	y	$^g/_k$	c^y	c^o	$1+r$	w
θ	$+$	$+$	$+$	$-$	$-$	$+$	$-$	$+$

Table 5.7: The effects of an increase in θ (general).

tor for old-age consumption. Therefore, the present young will, ceteris paribus, increase their savings. The increase of savings and, thus, of private capital, negatively affects the interest rate and positively affects the wage. The government, by increasing the level of public capital, positively affects the interest rate which benefits old-age consumption. However, when doing so it has to take into account that half of the extra investment is paid by the present elderly. This implies that the increase in g is smaller than the increase in k implying less efficiency. Of course, old-age consumption increases whereas young-age consumption decreases.

It can be concluded that, obliged to balance its budget each period and restricted to use time-dependent taxes only, the government will choose public capital levels which are below the bench-mark level, both in level and in relation to the level of private capital. The latter implies that the efficiency condition is not satisfied. The reason for these results is the impossibility of exempting the current elderly from paying one half of the investment. Their political weight negatively affects public investment. The high level of savings and, thus, of private capital can be explained from the relatively high wages and impossibility of giving a PAYG transfer to the current elderly. An increase in the relative political weight of the current young leads to more investment in public capital, also relative to the level of private capital, implying an efficiency gain with respect to the g/k-ratio. Besides, the consumption of the current elderly increases. If generations attach more weight to their old-age consumption, this implies, first of all, an increase of private savings. The level of public capital also increases, but less than the level of private capital because the government has to balance the positive effect of more public capital on old-age consumption of the present young against the negative effect of a higher τ^o for the current old. The result is a loss in efficiency of g relative to k.

5.4.2 A balanced-budget policy with time- and age-dependent taxes

Can the government improve upon the allocation of the previous section when it has the possibility to use time- and age-dependent taxes? In a world where the government maximizes social welfare, *i.e.* it takes the utilities of all present and future generations into account, the spillover to future generations is internalized and a system of time- and age-dependent taxes will suffice to obtain the command solution of a social welfare maximizer (see Calvo and Obstfeld (1988)). In the representative democracy setting this does not hold, however. The extra instrument gives the government the possibility to spread the burden of public capital more fairly over the two generations, *i.e.* taxing the young relatively more since they benefit from public capital. But the spillover is still not internalized. However, the undesired spillover to future generations can now be reduced by taxing generations differently and to invest less in public capital. But, this has negative consequences for the steady state.

In this case, the budget constraint, eq. (5.2.14), becomes

$$g_{t+1} = \tau_t^y + \tau_t^o \tag{5.4.4}$$

Note that a PAYG pension system might result since taxes are allowed to be negative. Using the budget constraint to eliminate τ_t^o, gives an optimization problem for the government in period t where it maximizes eq. (5.2.13) with respect to τ_t^y and g_{t+1}. Then, the first-order condition with respect to τ_t^y reads:

$$\theta \frac{\partial v_{t-1}}{\partial c_t^o} + \lambda \frac{\partial u_t}{\partial c_t^y} [-1] + \lambda\theta \frac{\partial v_t}{\partial c_{t+1}^o} \left[s_t \frac{d(1+r_{t+1})}{d\tau_t^y} \right] = 0 \tag{5.4.5}$$

where

$$\frac{d(1+r_{t+1})}{d\tau_t^y} = \frac{\frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2} \cdot \frac{\partial s_t}{\partial \tau_t^y}}{1 - \frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2} \cdot \frac{\partial s_t}{\partial(1+r_{t+1})}} \tag{5.4.6}$$

Taxing the young gives a positive effect on the wealth of the current elderly since higher taxes for the current young imply, ceteris paribus, lower taxes for the current elderly. This is given by the first term in eq. (5.4.5). The second term is the direct effect on the utility of the current young because taxation changes lifetime income. This is clearly negative. Finally[14], there is an income effect due to the change in the next period interest rate, which is given by eq. (5.4.6). The numerator denotes the effect which taxation of the young has on the marginal productivity of private capital and, thus, on the interest rate. For logarithmic preferences, τ_t^y has a negative effect on savings (which follows immediately from eq. (5.2.5)). Since marginal productivity of private capital declines with increasing k, less savings imply a higher marginal productivity. Hence, the numerator is

[14] Again, the substitution effect disappears.

positive. The denominator contains the multiplier effect of taxation. In case of logarithmic preferences, $1+r$ negatively affects savings. Thus, for logarithmic preferences and a Cobb-Douglas production technology, the denominator is positive and smaller than one, indicating a positive multiplier effect. Hence, $\frac{d(1+r_{t+1})}{d\tau_t^y}$ is positive.

The first-order condition with respect to g_{t+1} is given by:

$$\theta \frac{\partial v_{t-1}}{\partial c_t^o}\left[-1\right] + \lambda\theta \frac{\partial v_t}{\partial c_{t+1}^o}\left[s_t \frac{d(1+r_{t+1})}{dg_{t+1}}\right] = 0 \tag{5.4.7}$$

where

$$\frac{d(1+r_{t+1})}{dg_{t+1}} = \frac{\frac{\partial^2 y_{t+1}}{\partial k_{t+1}\partial g_{t+1}}}{1 - \frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2} \cdot \frac{\partial s_t}{\partial(1+r_{t+1})}} \tag{5.4.8}$$

The first term denotes the negative effect of public capital on the utility of the current elderly. The second term is the positive income effect for the current young[15]. The change in the interest rate causes this effect. Eq. (5.4.8) gives the effect of public capital on the interest rate. The numerator contains the direct effect of public capital on the marginal productivity of private capital. This is positive. The denominator contains the multiplier effect which is positive as well.

Again no analytical solutions can be derived. The equations determining the steady-state are given in the appendix. For further analysis simulations have to be used. For the baseline case, the results are given in table 5.8. Comparing these results to the

	g	k	y	g/k	τ^y	τ^o	U	c^y	c^o	
	0.231	1.226	7.771	0.189	1.270	-1.039	1.838	3.720	2.593	
(Benchmark:	1.043	1.043	10.170	1.000	—	—	2.231	5.215	2.869	)

Table 5.8: Baseline case if $g = \tau^y + \tau^o$.

bench-mark case and the results of the previous section, the following can be observed. The levels of private and public capital are lower. Production is therefore lower as well.

[15]See footnote 14

The g/k-ratio is still below its efficiency level and slightly lower than in the previous section. Consumption of the young is lower than in the bench-mark case and than in the previous section. Consumption of the elderly is also lower than in the bench-mark case but it is higher than in the previous section. How can these results be explained? Since the government optimizes eq. (5.2.13), a weighted function of both living generations, it may want to affect the intergenerational distribution of welfare between current generations. In the previous section it could only do this in a rather inefficient way. It had to tax both generations equally and had to use public capital to affect the intergenerational distribution of welfare. But, public capital has several effects which have to be taken into account, including a spillover to future generations. Here, compared to the previous section, the government has an extra instrument available: it can tax both present generations at a different level using a time- and age-dependent lump-sum tax. And, moreover, these instruments are non-distortionary. Therefore, it can redistribute more efficiently by taxing both generations differently and use less public capital for redistribution. As a result, public capital is lower than in the previous section. As can be seen in table 5.8, the tax system used operates as a PAYG pension system. The taxes paid by the young generation are mainly used to make a transfer to the elderly. This gives lower savings and, thus, a lower level of private capital. Note that utility is lower as well, not only compared to the bench-mark case but also compared to the previous section. This can be explained as follows. The effect of the lower level of public capital on future wage income is, of course, disregarded by the present government. However, this affects the steady-state levels. Thus, the lower levels of private and public capital lead, compared to the previous section, to a serious decline in the wage. As a result (and, of course, due to the disregarding of this effect), the steady-state utility level is lower. Compared to the bench-mark, the level of public capital is too low because the government is not allowed to use public debt in this section.

Just as in the previous section, changes in the steady-state levels due to changes in the exogenous parameters can be analyzed. The effects of an increase in λ are reported in table 5.9, which gives the variations around the baseline case, and in table 5.10 where other values of the other exogenous parameters are taken into account. Except for τ^y, all results are robust. Moreover the results are very similar to the results obtained in the previous section. With an exception, of course, for the taxes. More political weight for

	g	k	y	g/k	τ^y	τ^o	c^y	c^o	$1+r$	w
$\lambda = 0.8$	0.140	0.846	6.528	0.166	1.380	-1.240	2.997	2.545	1.540	5.222
1.0	0.231	1.226	7.771	0.189	1.270	-1.039	3.720	2.593	1.286	6.217
1.2	0.349	1.655	8.962	0.211	1.154	-0.805	4.360	2.597	1.083	7.170

Table 5.9: The effects of variations in λ around the baseline case.

	g	k	y	g/k	τ^y	τ^o	c^y	c^o	$1+r$	w
λ	+	+	+	+	$\pm$	+	+	+	$-$	+

Table 5.10: The effects of an increase in λ (general).

the present young implies an increase in taxes for the elderly. The government can give these extra tax revenues to the current young in two ways. First, by decreasing taxes for the current young generation. Second, by increasing public capital. In case production is rather inelastic with respect to public capital (*i.e.* β is low), a decrease in τ^y can be observed. If, however, public capital is more productive, more emphasis is given to public capital. As a result, τ^y can be seen to increase as well. Note that, as in the previous section, the current elderly benefit from an increase in political weight for the current young.

Tables 5.11 and 5.12 report the effects of changes in the private discount factor. From table 5.12 it follows that the results for the basline case hold for other values of the other exogenous parameters as well. The results are almost completely opposite to the

	g	k	y	g/k	τ^y	τ^o	c^y	c^o	$1+r$	w
$\theta = 0.40$	0.274	1.318	8.159	0.208	0.817	-0.543	4.392	2.174	1.238	6.527
0.55	0.231	1.226	7.771	0.189	1.270	-1.039	3.720	2.593	1.268	6.217
0.70	0.209	1.163	7.533	0.179	1.632	-1.424	3.231	2.930	1.295	6.026

Table 5.11: The effects of variations in θ around the baseline case.

results obtained in the previous paragraph. An increase in the private discount factor implies a stronger preference for old-age consumption. As before, this implies first-of-all

	g	k	y	g/k	τ^y	τ^o	c^y	c^o	$1+r$	w
θ	$-$	$-$	$-$	$-$	$+$	$-$	$-$	$+$	$+$	$-$

Table 5.12: The effects of an increase in θ (general).

an increase of savings and, thus, of private capital. Hence, there are two external effects. First, the effect on the interest rate. Second, the spillover to future generations. However, contrary to the previous paragraph, the present government can positively affect old-age consumption by raising τ^y and decreasing τ^o without all the external effects. But, then, the disregarding of the effect on future wages has strong negative consequences in the long run. Since public capital declines, future wages decline. This leads to a decline in savings. Therefore, in the long run, the level of private capital decreases.

Summarizing: Though the government has an extra instrument at its disposal, the allocation does not improve. On the contrary, the allocation worsens. The reason is that the government can now more efficiently affect the intergenerational distribution of welfare by taxing the generations differently. The government, thus, reduces the spillovers to future generations. However, this has serious consequences for the steady-state levels because the effect on future wages is much smaller. Compared to the previous section, consumption when young decreases while old-age consumption increases. Compared to the bench-mark case, consumption for both ages is lower. As in the previous section, the level of public capital increases if the current young generation becomes politically more important. Thus, efficiency with respect to the g/k-ratio improves. Contrary to the previous section, an increase in the private discount factor leads to lower public capital levels. As in the previous section, efficiency with respect to the g/k-ratio declines.

5.4.3 Debt

Not many countries balance their budget each period. On the contrary, public debt is widely used. For several reasons, many countries have restricted themselves in the use of public debt. E.g. in Germany the government is only allowed to borrow for public capital expenditures, with a maximum of 75% of these expenditures. Until World War II the Netherlands had a similar restriction on the use of public debt. Reestablishment of this norm has been discussed. The use of public debt gives the opportunity to spread the burden of public capital more equally over the different generations. The future young, who receive a higher wage income due to the investment in public capital, can now be confronted with a part of the costs. Thus, the spillover to future generations due to public investment can be 'compensated' by a spillover due to public debt. Present generations will take advantage of the fact that future generations are not represented in the current decision-making process. Therefore, the possibility of debt financing might lead to overinvestment in public capital if the costs of public investment for future generations exceed the benefits, which equal the spillover. Whether there will be overinvestment or not is, thus, closely connected to the extent in which present generations are able to avoid future debt repayments. It is assumed that the government is committed to repay the debt (including the interest obligations) during the depreciation period of the capital and that it is impossible for future governments to alter the repayment scheme or even completely repudiate the debt. In the present model, with full depreciation within one period, this leads to the following constraints for the government:

$$b_t = g_{t+1} \tag{5.4.9}$$

$$\tau^y_{t+1} = (1-\xi)(1+r_{t+1})b_t \tag{5.4.10}$$

$$\tau^o_{t+1} = \xi(1+r_{t+1})b_t \tag{5.4.11}$$

where ξ reflects the tax system and it denotes the part of taxes paid by the future elderly, *i.e.* the current young when old. When dealing with uniform taxes, ξ was equal to $1/2$. In the case of time- and age-dependent taxes, ξ was determined endogenously. Now, ξ is exogenous but allowed to vary between 0 and 1. It is assumed that ξ is smaller than 1. $1-\xi$ can be interpreted as the extent in which current generations succeed in shifting the costs of public investment onto the shoulders of future generations.

When deciding on a level of public investment, the current government takes the commitment of the next government to repay the debt into account and, thus, takes the effect of the current policy choice on the taxes to be paid by the present young when old into account. Then, the first-order condition determining the level of public capital is given by:

$$\frac{d(1+r_{t+1})}{dg_{t+1}}s_t = \xi(1+r_{t+1}) + \xi\frac{d(1+r_{t+1})}{dg_{t+1}}g_{t+1} \tag{5.4.12}$$

where

$$\frac{d(1+r_{t+1})}{dg_{t+1}} = \frac{\frac{\partial^2 y_{t+1}}{\partial k_{t+1}\partial g_{t+1}} + \frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2}\left[\frac{\partial s_t}{\partial \tau_{t+1}^o}\xi(1+r_{t+1}) - 1\right]}{1 - \frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2}\cdot\frac{\partial s_t}{\partial(1+r_{t+1})} - \xi\frac{\partial^2 y_{t+1}}{\partial k_{t+1}^2}\cdot\frac{\partial s_t}{\partial \tau_{t+1}^o}g_{t+1}} \tag{5.4.13}$$

The marginal revenues of an extra unit of public capital is given by the term on the left-hand-side in eq. (5.4.12). This is an income effect only since, as before, the substitution effect disappears. On the right-hand-side, the marginal costs consist of two parts. First, the direct effect on the next period interest rate (the first term on the right-hand-side). Second, public capital is financed with debt over which interest has to be paid. Since public capital affects the interest rate, this affects wealth (this is given by the second term). Note the difference with the previous sections. Because debt is used, the current old generation becomes irrelevant for the decision-making on public capital. Instead, future generations bear part of the burden of public capital. Since they are not represented in the current decision on public capital, this could lead to more public capital than the bench-mark level would require because present young will take advantage of the fact that future generations are not represented in the current decision-making process. Overinvestment, however, not necessarily occurs, it depends on the level of ξ.

Using the logaritmic specification of the utility function and a Cobb-Douglas production function, after tedious manipulations steady-state values can be derived. They are given in the appendix. However, for further results simulations have to be used. Tables 5.13 and 5.14 reports the effect of an increase in ξ on the steady-state levels. It follows from table 5.14 that the results are robust. First-of-all, note that the level of public capital may be above its bench-mark level. However, this depends not only on the value for ξ but also on the level of λ. For the model in this paragraph, the steady-state levels are independent of λ. But for the bench-mark case, they do depend on λ but are independent of ξ. Thus, in principle, for each value of ξ, there is a value of λ for which the level of public capital is identical. Comparing tables 5.2 and 5.13, it follows that utility levels

	g	k	y	g/k	U	c^y	c^o	$1+r$	w
$\xi = 0.755$	2.659	2.521	14.630	1.054	2.469	5.769	3.682	1.161	11.704
0.85	1.437	3.116	13.496	0.461	2.384	6.057	2.886	0.866	10.797
0.95	1.018	3.183	12.650	0.320	2.291	5.879	2.570	0.795	10.120
(Benchmark:	1.043	1.043	10.170	1.000	2.231	5.215	2.869	—	—)

Table 5.13: The effects of variations in ξ around the baseline case.

	g	k	y	g/k	c^y	c^o	$1+r$	w
ξ	−	+	−	−	$\pm$	−	−	−

Table 5.14: The effects of an increase in ξ (general).

can be below as well as above those in the bench-mark case. This also is caused by the independence of the decentralized solution of λ and the independence of the command solution of ξ. Comparing public capital levels or utility levels is therefore meaningless. It is more interesting to check whether there is over- or underinvestment with respect to the efficiency condition since this condition is independent of λ. Then it holds that, depending on ξ, there can be over- as well as underinvestment. In general, the g/k-ratio increases if ξ decreases because public capital becomes relatively more attractive than private capital and hence, a crowding-out of private capital results. For the bench-mark case, the efficiency condition is fulfilled if $\xi \approx 0.76$. Old-age consumption increases with a decline in ξ because, first, it implies that τ^o decreases and, second, the return on private savings increases since an increase in public capital and a decrease in private capital both lead to an increase in the interest rate. c^y first increases if ξ declines because wages increase due to the increase in production. However, a decline in ξ also implies an increase of τ^y. If ξ declines further this starts to dominate and c^y decreases. If ξ is sufficiently low, there is no steady state at all. In that case, compared to the costs, the benefits of public investment for the current young are so large that, eventually, public capital crowds-out all private capital. The following proposition gives a lower bound on the level of ξ:

Proposition 5.1 *The steady state exists if*

$$\xi > \frac{[(1-\alpha-\beta)(1-\beta+\frac{1-\alpha}{1+\theta})+2\beta]+\sqrt{4\beta[(1-\alpha-\beta)(\alpha+\frac{1-\alpha}{1+\theta})+\beta]}}{(1+\beta+\frac{1-\alpha}{1+\theta})^2-4\beta\frac{1-\alpha}{1+\theta}}$$

Proof: See the appendix.

The effects of an increase in the private discount factor are reported in tables 5.15 and 5.16. It implies, ceteris paribus, an increase in private savings. These private

	g	k	y	g/k	c^y	c^o	$1+r$	w
$\theta = 0.40$	0.921	2.239	11.557	0.411	5.943	2.454	1.032	9.246
0.55	1.437	3.116	13.496	0.461	6.057	2.886	0.866	10.797
0.70	1.985	3.863	15.029	0.514	5.944	3.238	0.778	12.023

Table 5.15: The effects of variations in θ around the baseline case ($\xi = 0.85$).

	g	k	y	g/k	c^y	c^o	$1+r$	w
θ	+	+	+	+	$\pm$	+	$-$	+

Table 5.16: The effects of an increase in θ (general).

savings are invested in private and public capital. Therefore, an increase in both can be observed. The effect more private capital has on the interest rate, which is not taken into account by the private sector, is treated in a similar way as in the case where the government could only use time-dependent taxes. The government increases public capital in order to compensate for this effect. However, in increase in public capital implies an increase in future taxes, also for the future young. This affects the steady state results since a higher τ^y negatively affects private savings. Besides, an increase in private savings does not imply an identical increase in private capital. A part of the savings is invested public debt and, thus, in public capital. Hence, the g/k-ratio increases. If θ is still relatively low, c^y increases as well because the increase in production. If θ increases further, c^y decreases however.

Concluding, allowing the government to use debt, with $b = g$, what are the effects for the level of public capital? If the costs associated with the capital could be sufficiently shifted onto the shoulders of future, yet unrepresented generations, *i.e.* ξ is small, over-investment in public capital relative to the level of private capital can result. If the share shifted onto the shoulders of future generations is too large, *i.e.* ξ is too small, there is no steady state at all. Public capital then crowds out all private capital. Not surprisingly, the level of public capital decreases with an increase in ξ. An increase in the private discount factor, implying a higher preference for old-age consumption, gives an increase in savings and, hence, an increase in private and public capital. Furthermore, note that changes in λ have no effect on the level of public or private capital. Since the current elderly are not relevant in the decision making on public capital, the relative political power is irrelevant.

5.5 Summary and concluding remarks

In this chapter, public investment is modelled as a conflict between present living generations. Present older generations are opposed to investment in public capital since they do not benefit from it. Younger generations have a higher rate of return on their old-age savings due to the investment and, therefore, are in favour of public investment. This conflict of interests is resolved in a representative democracy where the government is formed by the representatives of the present living generations. Thus, the government, when deciding on the level of public investment, only takes the utilities of current living generations into account where each generation is weighed according to its relative political influence.

Before analysing the resulting level of public investment, first, a bench-mark case is defined. Because of the form of the objective function and the fact that, in a representative democracy setting, the scope for policy making is confined to one period, this is not a trivial matter. It necessitated behavioural assumptions with respect to future dictators. From the command solution an efficiency condition is derived for g versus k. In the decentralized economy, the effects of investment in public capital on the utilities of the generations also depend on the way in which these invesments are financed. The command solution can be replicated in a market economy by using public debt and taxation of the young. The elderly receive a transfer then. However, this solution cannot be attained. Since the government took future policy as given, unrestricted use of public debt results in ever larger debt levels. Therefore, three different sets of policy instruments are analysed. In case the government has to balance its budget each period and has to tax both current generations at the same level, the chosen levels of public capital are below the bench-mark level and, moreover, the efficiency condition is not fulfilled because there is underinvestment in public capital relative to the level of private capital. The reason for this is the spillover to future generations whose utility is not taken into account by current government. Besides, the impossiblity of giving a transfer to the elderly leads to a high level of private saving and, thus, of private capital. If the government has the possibility to tax both generations differently but still has to balance its budget, the level of public capital is even lower than in the case where the government has to tax both generations identically. The reason for this result is that the government, having the possibility to tax both generations differently with a non-distortionary tax, can affect the intergenerational distribution of welfare more efficiently without the spillover to future generations. There is still investment in public capital, simply because it was neccessary for production, but it is much lower than before. The tax system in this case operates as a PAYG pension system giving a transfer to the elderly. As a result, private savings and, thus, private capital are lower as well. When the government is allowed to use debt, the resulting level of public capital depend on what share of the debt repayments have to be paid by current generations. If current generations are able to shift a large part onto future, currently unrepresented, generations, overinvestment in public capital relative to the level of private capital can result.

In the two balanced budget cases analyzed, the level of public capital depends positively on the weight attached by the government to the current young generation. Since they are the ones who benefit from investment in public capital, more political weight gives higher levels of public capital. Also efficiency improved. This result is robust. Note that in both balanced budget cases, the increased political weight is beneficial for the present elderly as well. In case of debt financing of public investment this political weight does not matter. Then, the extent to which future, yet unrepresented, generations can be confronted with the cost of the investment becomes important. The higher the share of these costs shifted on to the shoulders of these future generations, the higher the level of public investment. However, if this share is too high, there is no steady-state solution because public capital crowds out all private capital so that eventually there is no production at all.

The response of g to an increase in the private discount factor depends on the policy instruments available. An increase in the private discount factor implies a higher preference for old-age consumption. When the government can only use time-dependent taxes, this implies an increase in the level of public capital since this positively affects the rate of return on old-age savings. It has, however, a negative effect on efficiency. Because of the spillover to future generations, the increase in public capital was less than the increase in private capital. But in the case of time- and age-dependent taxes, the level of public capital decreases. An increase of taxes for the young and a decrease of taxes for the elderly is a more efficient way of increasing old-age consumption and decreasing young-age consumption because the government can avoid the spillover to future generations. Again the level of public capital relative to the level of private capital decreases, implying less efficiency. Finally when the government is allowed to use public debt, the results are similar to the balanced budget case with time-dependent taxes. With the exception that there is an increase in efficiency. The extra savings due to the increased preference for old-age consumption are, in this case, not fully absorbed by an increase in private capital but, partly, go to into an increase in public debt, and, thus, in public capital.

An analysis where three different sets of policy instruments are compared to a bench mark case begs the question which of these sets is to be preferred. This requires a comparison of the utility levels obtained. However, note that in the bench-mark case and in the two balanced budget cases, the steady-state levels and, thus, the utility levels depend on the value of λ whereas in the debt case they are all independent of λ. To settle the case is therefore not easy. Comparing the two balanced budget cases, for the steady state one can give a clear answer in favour of time-dependency of taxes only. The disregarding of the positive spillover to future wages has serious negative consequences for the steady-state levels in case the government can also discriminate according to age when choosing taxes. The independence of the solution for the debt case of λ and the independence of the solution for the bench-mark case and the balanced budget cases of ξ, implies that on the basis of utilities obtained the answer on the question which to prefer depends on values of both parameters (compare tables 5.2 and 5.13). Endogenizing ξ is therefore interesting but also complicated because there is an effect on the future interest rate and therefore on future interest payments on public debt and future interest revenues on private savings. A lower value of ξ implies more taxes to be paid by the current young which negatively affects private savings. This affects private capital levels and therefore the interest rate. Speculating somewhat, it is likely that ξ increases with λ, *i.e.* if the present young have more political weight, the share of debt repayments paid by the present elderly is larger. Then an increase in λ leads to an increase in ξ which, according to tables 5.13 and 5.14, leads to a decrease in steady-state utility levels. Besides utility levels, the efficiency condition can be taken for a comparison. Both balanced budget cases give almost identical results with respect to the efficiency condition. Efficiency may improve substantially if public debt is used. However, the level of public capital can become too high compared to the level of private capital. If ξ decreases with λ, then, if λ increases, efficiency improves if the budget is balanced, efficiency worsens if debt is used. However, more general conclusions, like for what levels of λ to use debt or taxes, depend, of course, on the exact relation between λ and ξ. For more general conclusions, further analysis is needed.

A Appendix

A.1 Steady state with time-dependent taxes

For logarithmic utility and Cobb-Douglas production the steady state in case of time dependent taxes is determined by the following equations:

$$k = \frac{{}^1\!/\!_2 g}{(1+\theta)\frac{\alpha y}{k}} + \frac{\theta}{1+\theta}[(1-\alpha)y - {}^1\!/\!_2 g] \tag{5.A.1}$$

$$\lambda k \frac{\frac{\alpha\beta y}{kg} + \frac{\alpha(1-\alpha)y}{k^2} \cdot \frac{\theta}{1+\theta} \cdot {}^1\!/\!_2}{1 - \frac{\alpha(1-\alpha)y}{k^2} \cdot \frac{{}^1\!/\!_2 \cdot g}{(1+\theta)\left(\frac{\alpha y}{k}\right)^2}} = {}^1\!/\!_2 \left(1 + \lambda \frac{\alpha y}{k}\right) \tag{5.A.2}$$

where y is given by eq. (5.2.10). Eq. (5.A.1) follows from inserting eqs. (5.2.15), (5.2.11), (5.2.12) and (5.4.1) into eq. (5.2.5), the optimal level of private savings. Eq.

(5.A.2) follows from eq. (5.4.2), the first-order condition of the government, and using eqs. (5.2.11), (5.4.1), (5.2.15) and (5.4.3).

A.2 Steady state with time- and age-dependent taxes

For the logarithmic utility function and the Cobb-Douglas production function the steady state in case of time- and age-dependent taxes is determined by the following equations:

$$k = \frac{g - \tau^y}{(1+\theta)\frac{\alpha y}{k}} + \frac{\theta}{1+\theta}[(1-\alpha)y - \tau^y] \tag{5.A.3}$$

$$\lambda k \frac{\frac{\theta}{1+\theta} \cdot \frac{\alpha(1-\alpha)y}{k^2}}{1 - \frac{\alpha(1-\alpha)y}{k^2} \cdot \frac{g-\tau^y}{(1+\theta)\frac{\alpha y}{k}}} = 1 - \lambda \frac{\alpha y}{k} \tag{5.A.4}$$

$$\lambda k \frac{\frac{\alpha \beta y}{kg}}{1 - \frac{\alpha(1-\alpha)y}{k^2} \cdot \frac{g-\tau^y}{(1+\theta)\frac{\alpha y}{k}}} = 1 \tag{5.A.5}$$

where y is given by eq. (5.2.10). Eq. (5.A.3) follows from inserting eqs. (5.2.11), (5.2.12), (5.2.15) and (5.4.4) in eq. (5.2.5). Eq. (5.A.4) follows from using eqs. (5.2.11), (5.2.15), (5.4.4) and (5.4.6) in eq. (5.4.5). Eq. (5.A.5) follows from eq. (5.4.7) and using eqs. (5.2.11), (5.2.15), (5.4.4) and (5.4.8).

A.3 Steady state with debt

Using the logaritmic specification of the utility function and the Cobb-Douglas production function, after tedious manipulations the following steady-state values for g and k can be derived:

$$g = \left\{ \frac{\frac{\theta}{1+\theta}[(1-\alpha) - \alpha(1-\xi)\frac{g}{k}]A\frac{g}{k}^{\beta}}{1 + (1 - \frac{\xi}{1+\theta})\frac{g}{k}} \right\}^{\frac{1}{1-\alpha-\beta}} \frac{g}{k} \tag{5.A.6}$$

$$k = \left\{ \frac{\frac{\theta}{1+\theta}[(1-\alpha) - \alpha(1-\xi)\frac{g}{k}]A\frac{g}{k}^{\beta}}{1 + (1 - \frac{\xi}{1+\theta})\frac{g}{k}} \right\}^{\frac{1}{1-\alpha-\beta}} \tag{5.A.7}$$

where

$$\frac{g}{k} = \frac{-[(1-\alpha)(1-\frac{\xi}{1+\theta}) + \beta(1-\xi) - \xi]}{2(1-\xi)(1-\frac{\xi}{1+\theta})(1-\alpha)} -$$

$$- \frac{\sqrt{[(1-\alpha)(1-\frac{\xi}{1+\theta}) + \beta(1-\xi) - \xi]^2 - 4\beta(1-\alpha)(1-\frac{\xi}{1+\theta})(1-\xi)}}{2(1-\xi)(1-\frac{\xi}{1+\theta})(1-\alpha)} \quad (5.A.8)$$

The steady-state value for y can be found by substituting the expressions for g and k into the production function eq. (5.2.10). As noted in Proposition 5.1, these steady-state values are only defined if ξ is sufficiently high.

A.4 Proof of Proposition 1

Define $\psi = \frac{g}{k}$. Then the first-order condition of the government, eq. (5.4.12) can be written as:

$$\beta + [\beta - \xi\beta - \xi + (1-\alpha)(1-\frac{\xi}{1+\theta})]\psi + (1-\xi)(1-\alpha)(1-\frac{\xi}{1+\theta})\psi^2 = 0$$

Solving this for ψ gives:

$$\psi = \frac{-[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]}{2(1-\xi)(1-\frac{\xi}{1+\theta})(1-\alpha)} \pm$$

$$\pm \frac{\sqrt{[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]^2 - 4\beta(1-\alpha)(1-\frac{\xi}{1+\theta})(1-\xi)}}{2(1-\xi)(1-\frac{\xi}{1+\theta})(1-\alpha)} \quad (5.A.9)$$

From the second-order condition for the government's problem, which is given by

$$2(1-\xi)(1-\alpha)(1-\frac{\xi}{1+\theta})\psi\frac{d\psi}{dg} + [(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]\frac{d\psi}{dg}$$

where $\frac{d\psi}{dg} = \frac{1}{k} - \frac{g}{k^2}(\frac{\xi}{1+\theta} - 1) > 0$ since, by assumption, $\xi < 1$, it follows that

$$\psi < -\frac{[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]}{2(1-\xi)(1-\alpha)(1-\frac{\xi}{1+\theta})} \quad (5.A.10)$$

Combining eqs. (5.A.9) and (5.A.10) gives:

$$\psi = \frac{-[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]}{2(1-\xi)(1-\frac{\xi}{1+\theta})(1-\alpha)} -$$

$$- \frac{\sqrt{[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]^2 - 4\beta(1-\alpha)(1-\frac{\xi}{1+\theta})(1-\xi)}}{2(1-\xi)(1-\frac{\xi}{1+\theta})(1-\alpha)} \quad (5.A.11)$$

Furthermore, it must hold that $\psi > 0$ and the square root in eq. (5.A.11) must be positive.

1. $\psi > 0$

From the second-order condition follows

$$\psi < -\frac{[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]}{2(1-\xi)(1-\alpha)(1-\frac{\xi}{1+\theta})}$$

Hence, since $\psi > 0$, $(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi < 0$. This implies

$$\xi > \frac{1-\alpha+\beta}{1+\beta+\frac{1-\alpha}{1+\theta}} \quad (5.A.12)$$

Thus, since $4\beta(1-\alpha)(1-\frac{\xi}{1+\theta})(1-\xi) > 0$, it holds that $\psi > 0$.

2. $[(1-\alpha)(1-\frac{\xi}{1+\theta}) + (1-\xi)\beta - \xi]^2 - 4\beta(1-\alpha)(1-\frac{\xi}{1+\theta})(1-\xi) > 0$

This expression can be rewritten as

$$[(1+\beta+\frac{1-\alpha}{1+\theta})^2 - 4\beta\frac{1-\alpha}{1+\theta}]\xi^2 -$$

$$- 2[(1-\alpha-\beta)(1-\beta+\frac{1-\alpha}{1+\theta}) + 2\beta]\xi + (1-\alpha-\beta)^2 > 0 \quad (5.A.13)$$

The roots are given by

$$\xi = \frac{[(1-\alpha-\beta)(1-\beta+\frac{1-\alpha}{1+\theta}) + 2\beta] \pm \sqrt{4\beta[(1-\alpha-\beta)(\alpha+\frac{1-\alpha}{1+\theta}) + \beta]}}{(1+\beta+\frac{1-\alpha}{1+\theta})^2 - 4\beta\frac{1-\alpha}{1+\theta}}$$

Since $(1+\beta+\frac{1-\alpha}{1+\theta})^2 - 4\beta\frac{1-\alpha}{1+\theta} = (1+\beta)^2 - (1-\beta)^2 + [1-\beta+\frac{1-\alpha}{1+\theta}]^2 > 0$, eq. (5.A.13) has a minimum, and, hence, the right-hand-side of eq. (5.A.12) is smaller than

$$\frac{[(1-\alpha-\beta)(1-\beta+\frac{1-\alpha}{1+\theta})+2\beta]+\sqrt{4\beta[(1-\alpha-\beta)(\alpha+\frac{1-\alpha}{1+\theta})+\beta]}}{(1+\beta+\frac{1-\alpha}{1+\theta})^2-4\beta\frac{1-\alpha}{1+\theta}}$$

It must hold that

$$\xi > \frac{[(1-\alpha-\beta)(1-\beta+\frac{1-\alpha}{1+\theta})+2\beta]+\sqrt{4\beta[(1-\alpha-\beta)(\alpha+\frac{1-\alpha}{1+\theta})+\beta]}}{(1+\beta+\frac{1-\alpha}{1+\theta})^2-4\beta\frac{1-\alpha}{1+\theta}}$$

Q.E.D.

Chapter 6

Final words

In this thesis, conflicts between present generations over redistributive public policies are analysed. These conflicts arise because the generations involved have different preferences over these policies. The decisions on these policies are made by the government. Thus, when analyzing redistributive policies from a more positive point of view, the political decision-making has to be taken into account. Most of the public choice literature assumes direct democracies. But since direct democracies are unusual in reality, the value of this literature for describing actual government policy is limited. Most countries are representative democracies where the government is formed by politicians who are elected to represent the interests of their voters. While in office, the politicians are under constant pressure from interest groups to enact those policies favoured by these interest groups. The extent to which these groups get their preferred policies enacted depends on their political importance. Policies chosen therefore are a political compromise between the preferences of these groups. There are, however, relatively few studies analysing the effects on the economy of political decision-making in a representative democracy.

In this thesis, redistribution between generations is modelled as the outcome of a political conflict in a representative democracy. The politicians are assumed to represent the interests of the current, non-altruistic generations only. These generations are supposed to live for two periods, during which they are labelled 'young' and 'old' respectively. Hence, a period in the model corresponds to a time span of approximately thirty years. By assuming individuals (and politicians) to be non-altruistic, future generations' utility is disregarded in policy decisions. The analysis is restricted to three types of policies because these are considered typical for most redistributive public policies. These types are: public debt, public pensions and public investment. For an extensive summary of the results the reader is referred to the sections concluding the chapters dealing with each of these policies. The remainder of this chapter focuses on the non-standard part of this thesis, the representative democracy. Besides, some ideas for further research are given.

Representative democracy. Modelling the government in a representative democracy framework has some interesting consequences when a dynamic model is used since the term in office of a government is confined to one period[1]. Each period the incumbent government is replaced by a new one, chosen by the new constituency. However, the current young generation is still present in the next period. Thus, future policies affect the utility of this generation. This implies that behavioural assumptions with respect to these future policy choices are necessary. Two possibilities can be distinguished. Either future policy is taken as given (Nash behaviour) or future reactions on current policy choices are taken into account (Stackelberg behaviour). Because of the sequential order in which the decisions of the different governments are taken, Stackelberg behaviour is the most natural assumption. However, it is well-known that Stackelberg models become insoluble analytically if production and utility functions are only slightly more complicated than a linear or quadratic form. But, when analysing public debt policies, as is done in Chapter 3, in the absence of altruism, the assumption of Stackelberg behaviour is quite crucial because it prevents current generations from issuing the largest sustainable level of public debt. Therefore, if the assumption of Stackelberg behaviour is made, the analysis is often confined to two periods (*e.g.* Persson and Svensson (1989), Tabellini and Alesina (1990) or Tabellini (1991)). However, the drawback of a two period analysis is that the solution may be strongly affected by end-of-time effects making the analysis of dynamics impossible. Since the public debt has to be redeemed the second period, policy in this final period is to a large extent determined beforehand. Besides, in these two-period models each individuals horizon is equivalent and there is no inherent tendency to shift taxes to future periods as these future taxes have to be paid by the individuals themselves. Thus, the possibility for (some) generations to avoid the burden of debt is absent. In Chapter 4, dealing with PAYG public pensions, Stackelberg behaviour is the

[1]In reality elections are held more frequently than once every thirty years, which could lead to a change in government. However, as the constituency nor its preferences change during a period (of thirty years), this will not happen in the models in this thesis.

point of departure but a comparison to Nash behaviour is made as well. In Chapter 5, however, Stackelberg behaviour is replaced by the assumption of Nash behaviour to keep the analysis somewhat tractable.

Two comparisons suggest themselves. First, to study the effects of the disregarding of future generations, the policy chosen by the representative government can be compared to the policy chosen by a social welfare maximizer, *i.e.* a government that not only takes the utility of present generations into account but of future generations as well. The second comparison is between the representative government in the market economy and its command counterpart. This comparison reveals which instruments are needed to obtain (if possible) the allocation chosen in a command economy, *i.e.* an economy where a dictator can directly choose consumption and capital levels. However, when defining a command solution to serve as a bench-mark case, some conceptual problems arise. Usually a command solution is derived by a dictator maximizing a social welfare function where the utility of all present and future generations is taken into account. But then, the following two points should be noted. First, in the representative democracy setting as modelled in this thesis, only the utility of current generations is taken into account. The utility of future generations is completely neglected. A social welfare maximizing dictator does take the utility of these future generations into account. Second, in a social welfare function, future generations are weighed with a social discount factor, which has to be smaller than 1, if a maximum is to be well defined. The political weight attached to the present young generation in a representative democracy framework, can, in principle, take any value. These two points show that the difference between the command and the decentralized allocation might also stem from the difference in the objective function. The alternative is comparison to a myopic dictator that only takes present generations into account and, thus, has the same objective function as the representative government in the market economy. Because some generations are still present when the current myopic dictator is replaced by a new myopic dictator, future policy choices matter for these generations. More concrete, for the two-overlapping-generations model used in this thesis, the current young are present the next period as well. Thus, their old-age consumption appears in the objective function of the current myopic dictator as well as of the next-period myopic dictator. Since the next-period myopic dictator actually sets this consumption level, like the current myopic dictator sets the consumption level of the current elderly, behavioural assumptions with respect to the next-period myopic dictator are necessary. Again, Stackelberg behaviour is the most natural assumption.

In this thesis, comparisons to a social welfare maximizing government as well as to a myopic dictator are made. However, in Chapters 3 and 4, only a comparison is made to a social welfare maximizing government. After this comparison, the comparison to a command solution is less interesting because the command solution of a social welfare maximizer coincides with the decentralized solution of a social welfare maximizer. The simplicity of the models used in these chapters makes the sets of instruments available sufficient to obtain this result. In Chapter 5, a command solution is used as a bench-mark case. Thus, the focus is on sufficiency of the available instruments to obtain this bench-mark solution. This does not imply, however, that the comparison to a decentralized social welfare maximizer is not interesting in this case. On the contrary, the disregarding of future generations has serious consequences. As investment in public capital is productive with a lag of one period in this chapter, current investment affects future interest rates, which is beneficial to the current young, and future wages, which benefits only future generations and none of the current generations. The present government regards this effect on future wages as a spillover in which it has no interest and prefers the spillover to be as small as possible. It is only interested in these future generations insofar it can confront these generations with the costs of current policy choices. Taking the utility of future generations into account implies internalizing this spillover. If such a social welfare maximizing government can use time- and age-dependent taxes, it can even replicate the command solution of a social welfare maximizer (comparable to the result of Calvo and Obstfeld (1988)). This contrasts with the result for the representative government, when the availability of time- and age-dependent taxes leads to a change for the worse. Also the possibility of using public debt would not lead to a total crowding-out of private capital by public capital, as the case may be in Chapter 5, if the utility of future generations is taken into account, because the negative effect on their utility of their share of the debt repayments is taken into account as well.

A few results are worth mentioning again. In the models used here, myopia of politicians, *i.e.* the disregarding of future generations, is not necessarily 'bad'. Compared to a social welfare maximizing government, the representative government in Chapters 3 and 4 may choose identical policies if the political weight of the current young happened to be sufficiently high. Despite the absence of altruism towards future generations, the fact that the current young have one extra period to live and take the effect of current policy choices on future policy choices into account, implies that politicians may act *as if* future generations are taken into account. A sufficiently high political weight for the present young generation compensates for the disregarding of future generations. Especially when dealing with public debt, if there is no altruism, the assumption of Stackelberg behaviour prevents current generations from issuing the largest sustainable level of public debt. Still, one would expect debt levels to increase in the course of time. But this does not have to hold either, even debt levels that decrease in the course of time may result. A similar result is obtained for PAYG public pensions which also may decrease in the course of time.

Chapter 5 deals with investment in public capital. Different sets of policy instruments are analysed and compared to the command solution for a representative government. When the government is restricted to balance its budget each period, an inefficient low level of public capital relative to the level of private capital results. The political influence of the current elderly, who have nothing to gain from investment in public capital, is an important force behind this result. However, if the current elderly can be exempted from taxation, because taxes are time- and age-dependent, even lower levels of public capital result. By taxing both generations differently, the government can affect the distribution of welfare between the two generations more efficiently without the large spillover to future generations. However, a smaller spillover has strong negative consequences for the steady state allocation because the lower future wage depresses future savings and, thus, future capital and production levels. When the government is allowed to use debt, it can confront the future generations with a share of the costs of public investment. Thus, the spillover to these future generations in the form of higher wages is 'compensated' by a spillover in the form of more public debt to be repayed. However, this not necessarily leads to overinvestment in public capital. This depends on the extent to which the current young can avoid future debt repayments. Again, disregarding future generations in the current decision-making process, does not have to lead to a complete exploitation of these future generations.

Further extensions. The homogeneity of the individuals of the same generation, the absence of altruism, the assumption that politicians only care about their voters' preferences and have no preferences of their own (including a long-term view based on some ideology) or the exogeneity of the political weights, to name but a few, are strong assumptions. The simplicity of the models used enables to derive analytical solutions describing the evolution of public debt and public pensions respectively and to keep the analysis on public investment tractable. But reality is more complex. In this thesis, the political weights are exogenous. What determines the success of interest groups in obtaining influence on the decisions of the government, *i.e.* this political weight, is, however, an important question. There are studies in which the political weight is endogenized. *E.g.* Coughlin et al. (1990) uses probabilistic voting models and assumes that politicians maximize votes. But this seems to be an oversimplified view on the behaviour of politicians. However, some of the explanatory factors, like size and homogeneity of an interest group, are likely to be important. More realistic are models based on asymmetries of information between politicians and interest groups. Politicians rely on information from specific interest groups for their policy decisions. Interest groups will therefore try to steer the policy decisions in their preferred direction by providing the 'right' information to the politicians (*e.g.* Potters and Van Winden (1993)). Also, endowing the models in Chapters 3 and 4 with a richer economic structure is certainly preferable. *E.g.* more heterogeneity between individuals and a more detailed production structure.

Part of the literature on endogenous growth is based upon public investement (*e.g.* Barro (1990), Glomm and Ravikumar (1994a, 1994b)). Linking the literature of endogenous growth to political decision making on public capital is therefore interesting. In Alesina and Rodrik (1994) this is done by assuming that the decision on the level of public capital is taken in a direct democracy where voters differ in their labour-capital ratio. This leads to overinvestment in public capital relative to the growth maximizing level. Investment in public capital is financed by a tax on capital income. The growth maximizing tax rate is the tax rate preferred by a pure capitalist, *i.e.* an individual with only capital income and no labour income. The less capital income the median voter has, the higher the tax rate preferred, because more public capital raises labour income which benefits the median voter more if he has relatively more labour income.

This implies too much investment in public capital relative to the growth maximizing level. It would be interesting to analyse the effect on growth if the decision is taken in a representative democracy. This implies extending the model in Chapter 5 to allow for endogenous growth. The conjecture is that, compared to the maximum level of growth attainable, a lower level of growth results because of too little investment in public capital. The political weight of the current elderly depresses the level of public capital if they have to pay a part of the costs. But also the current young do not prefer a growth maximizing level because of the spillover to future generations. They will therefore only support the growth maximizing level of public capital if they can shift an adequate part of the costs onto the shoulders of the present elderly or of the future generations by using public debt. But this can also lead to an overinvestment in public capital compared to the growth maximizing level. Further analysis is therefore very interesting.

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Samenvatting

Om diverse redenen grijpen overheden in in het economische proces. Veel overheidsingrijpen herverdeelt goederen en middelen tussen generaties. Deze intergenerationele herverdelingen zijn het onderwerp van dit proefschrift. Herverdelingen worden niet alleen ingegeven door onvoldoende marktwerking of ongewenstheid van een allocatie maar ook, afhankelijk van het politieke systeem, door preferenties van politici zelf, pressie van belangengroepen in de samenleving, etcetera. Er bestaan verschillende politieke systemen waarin overheden opereren. De meeste westerse landen zijn representatieve democratieën. Dit systeem is daarom het uitgangspunt van deze studie. De samenleving bestaat uit vele groepen met conflicterende belangen die trachten hun stempel op het overheidsbeleid te drukken door die politici te kiezen die het meest voor hun belangen opkomen. Daarnaast oefenen ze voortdurend druk uit op de politici om het gevoerde beleid in de door hen geprefereerde richting te sturen.

De studie wordt ingeleid in Hoofdstuk 1 waarin het bovenstaande in meer detail wordt besproken. Daarnaast worden drie bronnen van intergenerationele herverdeling die in deze studie geanalyseerd worden, te weten overheidsschuld, overheidspensioenen en overheidsinvesteringen, geïntroduceerd. Deze zijn kenmerkend voor een groot deel van de intergenerationele herverdeling door de overheid.

Hoofdstuk 2 gaat in op het modelleren van het intergenerationele conflict. Eerst worden de bouwstenen gegeven zoals overlappende-generatie modellen en speltheorie. De individuele leden van de verschillende generaties zijn niet altruïstisch verondersteld. Dit leidt ertoe dat Ricardiaanse Equivalentie niet geldt en het schuld- en belastingbeleid van de overheid reële effecten heeft. De aanname van een representatieve democratie heeft belangrijke consequenties. Omdat iedere periode het electoraat verandert, is er iedere periode een nieuwe overheid, gekozen door het nieuwe electoraat. De beleidskeuzes van

de toekomstige overheid kunnen echter gevolgen hebben voor (een deel van) het huidige electoraat. Gedragsaannames van de huidige overheid ten opzichte van de beleidskeuzes van toekomstige overheden zijn daarom nodig. Hiervoor zijn twee mogelijkheden: Nash of Stackelberg. In geval van Nash gedrag worden toekomstige beleidskeuzes als gegeven genomen; in geval van Stackelberg gedrag worden de toekomstige reacties op huidige beleidskeuzes meegenomen bij de keuze van het huidige beleid. Gegeven de volgorde in de tijd waarin de verschillende overheden opereren, ligt Stackelberg gedrag het meest voor de hand. In Hoofdstuk 3, handelend over overheidsschuld, en in Hoofdstuk 4, handelend over overheidspensioenen, is Stackelberg gedrag ten opzichte van toekomstige overheden aangenomen. De complexiteit van het model in Hoofdstuk 5 noodzaakt echter de aanname van Nash gedrag in plaats van Stackelberg gedrag. Een ander verschil is dat in dit hoofdstuk de analyse beperkt wordt tot stationaire toestanden. In de Hoofdstukken 3 en 4 wordt daarentegen het hele tijdspad geanalyseerd.
Twee vergelijkingen zijn mogelijk. Ten eerste de vergelijking met een overheid die een sociale welvaartsfunctie optimaliseert waarin niet alleen het nut van huidige generaties vertegenwoordigd is maar ook dat van toekomstige generaties. Met deze vergelijking kan het effect van het negeren van toekomstige generaties geanalyseerd worden. De tweede vergelijking is die met een centrale planner die direct consumptienivo's kan kiezen. In dit geval wordt geanalyseerd welke instrumenten de overheid nodig heeft om deze centrale planner allocatie via de markt te bereiken. De eerste vergelijking wordt gemaakt in de Hoofdstukken 3 en 4, de tweede in Hoofdstuk 5.

Hoofdstuk 3 analyseert de effecten van het intergenerationele conflict op de evolutie van overheidsschuld. Het conflict is als volgt: oudere generaties prefereren een zo hoog mogelijk schuldnivo omdat zij, als de schuld afbetaald moet worden, er niet meer zijn en dus geen last ondervinden van de hiervoor noodzakelijke belastingverhoging. Omdat de schuld niet eeuwig doorgeschoven kan worden, de binnen- en buitenlandse beleggers zullen dit voorkomen, zullen de jongeren wel geconfronteerd worden met een toekomstige belastingverhoging. Vanwege de preferentie consumptie gelijkmatig over hun leven te spreiden, zullen zij nu een lager nivo van schuld prefereren dan de ouderen omdat dan de toekomstige belasting lager kan blijven. Het pad dat het schuldnivo volgt door de tijd is afhankelijk van de parameters van het model, te weten de bevolkingsgroeivoet, de rentestand, het relatieve politieke gewicht van een jongere ten opzichte van een oudere en de discontofactor van de individuen. Afhankelijk van deze parameters zal een stijgend of een dalend nivo van schuld over de tijd resulteren. In geval van een stijgende schuld convergeert deze naar een maximum nivo wat bepaald wordt door de contante waarde van de huidige en toekomstige belastingbasis minus de contante waarde van huidige en toekomstige overheidsuitgaven. De vergelijking met een overheid die een sociale welvaartsfunctie optimaliseert en niet alleen het nut van huidige generaties maar ook dat van toekomstige generaties meeneemt levert het volgende op: ieder tijdspad dat gekozen wordt door een representatieve overheid kan ook gekozen worden door een overheid die toekomstige generaties meeneemt. Met andere woorden, een representatieve overheid kan zich gedragen alsof ze toekomstige generaties meeweegt in haar besluitvorming. Het omgekeerde geldt echter niet; er zijn dus tijdspaden die wel door een overheid die sociale welvaart maximaliseert gekozen kunnen worden maar niet door een representatieve overheid. Tot slot zijn in dit hoofdstuk geanticipeerde en ongeanticipeerde schokken op de exogene parameters geanalyseerd.

Hoofdstuk 4 behandelt de evolutie van een Pay-As-You-Go pensioen systeem. In een Pay-As-You-Go pensioen systeem worden huidige pensioenen gefinancierd uit huidige belastingopbrengsten. Met andere woorden, het pensioen van de huidige ouderen wordt betaald door de huidige jongeren. Oudere generaties zullen trachten, door middel van hun politieke invloed, een zo hoog mogelijk pensioen af te dwingen. Jongeren, daarentegen, willen een zo klein mogelijk pensioen betalen aan de ouderen. Liever nog zouden zij een overdracht van de ouderen afdwingen. Vergelijkbaar met het vorige hoofdstuk, zal er, afhankelijk van de bevolkingsgroeivoet, de rentevoet, het relatieve politieke gewicht van de jongeren en de discontofactor van de individuen, een dalend of stijgend nivo van pensioenoverdrachten resulteren. Vervolgens is afgeleid wanneer huidige generaties erop vooruitgaan in termen van nut bij de introductie van een Pay-As-You-Go systeem. Daarna wordt een vergelijking gemaakt tussen Nash gedrag en Stackelberg gedrag met betrekking tot toekomstige overheden. Dit leidt tot de conclusie dat het pensioennivo onder Nash gedrag altijd lager is dan het pensioennivo onder Stackelberg gedrag bij een gelijke uitgangspositie. Voor het nivo van besparingen geldt het tegenovergestelde. Dit verschil wordt veroorzaakt door het positieve effect dat de huidige pensioenoverdracht heeft op de toekomstige pensioenoverdracht. In geval van Nash gedrag wordt dit effect niet meegenomen wat resulteert in een hoger spaarnivo en een lagere pensioenoverdracht.

Uit de vergelijking met een overheid die een sociale welvaartsfunctie optimaliseert waarin ook de nutten van toekomstige generaties zijn meegewogen volgt dat ieder pad gekozen door een representatieve overheid ook gekozen kan worden door een overheid die toekomstige generaties meeweegt. Dit resultaat verschilt met dat in het vorige hoofdstuk. Dit verschil wordt veroorzaakt door het feit dat pensioenen zorgen voor een herverdeling tussen generaties binnen een periode terwijl schuld herverdeelt tussen generaties over periodes. Verder is afgeleid wanneer, voor een gegeven nivo van besparingen, huidige generaties beter af zijn met een representatieve overheid dan met een overheid die sociale welvaart maximaliseert. Tot slot zijn de effecten van veranderingen in de exogene parameters geanalyseerd.

Het intergenerationele conflict met betrekking tot overheidsinvesteringen wordt behandeld in Hoofdstuk 5. Omdat investeringen produktief zijn met een vertraging van één periode zijn huidige ouderen niet geïnteresseerd in deze investeringen. Jongeren daarentegen zijn wel geïnteresseerd omdat investeringen een positief effect hebben op de opbrengst van hun besparingen. Daarnaast veroorzaken investeringen ook nog een 'spill-over' naar toekomstige generaties omdat het looninkomen van deze generaties positief beïnvloed wordt. De oplossing van het model voor een centrale planner dient als referentiepunt. Uit deze oplossing volgt een voorwaarde voor efficiëntie, namelijk dat de marginale produktiviteit van particulier kapitaal gelijk is aan de marginale produktiviteit van overheidskapitaal. Om deze oplossing met behulp van marktinstrumenten te bereiken is een belasting voor de jongeren nodig. Daarnaast wordt schuld gebruikt. De inkomsten aldus verkregen worden gebruikt voor een overdracht naar de ouderen en voor investeringen in overheidskapitaal. Echter deze oplossing kan niet bereikt worden. De reden is dat in dit hoofdstuk de overheid toekomstig beleid als gegeven neemt (Nash gedrag). Ongelimiteerd schuldgebruik zal dan leiden tot een maximaal gebruik van schuld. Daarom zijn restricties gelegd op de beschikbare beleidsinstrumenten voor de overheid. Drie situaties zijn geanalyseerd. De eerste situatie betreft een overheid die haar begroting iedere periode sluitend moet houden en investeringen moet financieren met behulp van een uniforme 'lump-sum' belasting. Beide generaties betalen dus hetzelfde bedrag aan belasting. Het nivo van overheidsinvesteringen is lager dan het referentienivo. Ook in verhouding met het nivo van particuliere investeringen, is het nivo van overheidsinvesteringen te laag. Een tweetal redenen kunnen hiervoor gegeven worden. Ten eerste is er de spill-over

naar toekomstige generaties waarin geen van de huidige generaties in geïnteresseerd is. Ten tweede betalen de huidige ouderen de helft van de investeringen hoewel ze er geen enkele baat bij hebben. Dat de ouderen niet ontzien kunnen worden leidt tot een te hoog nivo van besparingen en daardoor van particulier kapitaal. In de tweede situatie kan de overheid beide generaties verschillend belasten maar moet ze haar begroting nog steeds sluitend houden. Het gevolg is een nivo van overheidsinvesteringen dat zelfs lager is dan bij een uniforme belasting. De reden voor dit resultaat is dat door de mogelijkheid generaties verschillend te belasten, de overheid de intergenerationele verdeling van welvaart efficiënter kan beïnvloeden dan bij een uniforme belasting. Bij een uniforme belasting moest ze gebruik maken van overheidsinvesteringen die een ongewenste spill-over veroorzaakten naar toekomstige generaties. Nu kan ze die spill-over vermijden. Het belastingsysteem werkt in dit geval als een soort Pay-As-You-Go pensioen systeem. Dit leidt ertoe dat ook het nivo van particulier kapitaal lager is dan bij uniforme belastingen. In de derde situatie is een soort kapitaaldienst verondersteld. De overheid mag lenen voor investeringsuitgaven maar is verplicht de aangegane leningen terug te betalen gedurende de afschrijvingsperiode van de investering. Het investeringsnivo in overheidskapitaal is in dit geval sterk afhankelijk van de mate waarin de lasten afgewenteld kunnen worden op toekomstige generaties. Wanneer deze afwenteling voldoende hoog is, kan een inefficiënt hoog nivo van overheidskapitaal het resultaat zijn.
Een stijging van de politieke invloed van de jongeren leidt in de situaties met een sluitende begroting tot een stijging van het nivo van overheidsinvesteringen. Ook een meer efficiënte inzet van overheidskapitaal resulteert. Bij de kapitaaldienst speelde deze politieke invloed geen rol. Immers door het gebruik van schuld worden de huidige ouderen niet meer geconfronteerd met de lasten van de huidige investeringen. Een stijging van de discontovoet van de individuen betekent meer gewicht voor consumptie in de tweede periode. In geval van uniforme belasting betekent dit een toename van de hoeveelheid overheidskapitaal. Echter, besparingen, en daarmee particulier kapitaal stijgt sterker wat minder efficiëntie betekent. In geval van een leeftijdsafhankelijke belasting daalt het nivo van overheidskapitaal. In geval van schuldfinanciering, tenslotte, stijgt het nivo van overheidskapitaal. Bovendien verbetert in dit geval de efficiëntie.

Hoofdstuk 6 vat de belangrijkste inzichten die dit proefschrift oplevert samen en geeft een aantal aanzetten voor verder onderzoek.

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MARTIJN VAN DE VE[...] in 1990. Since then he has been affiliated with the department of Economics at this university. His PhD research dealt with political decision making and intergenerational redistribution. Since April 1, 1996, he is affiliated with Nyfer Forum for Economic Research.

This thesis deals with political decision making on intergenerational redistributive policies via public debt, public pensions and public investment. Members of different generations have different preferences on these redistributive policies. The resulting intergenerational conflict is resolved in the political process. Using a representative democracy framework, the decision making on these policies is performed by politicians who are chosen to represent the different generations present. Future generations not yet alive are therefore not represented though they may be confronted with the effects of current policy choices. The evolution of public policies due to this political conflict is analysed where the effect of current policy choices on future policy choices is taken into account. Besides, comparisons are made to social welfare maximizing governments and central planners. Also the effects of different sets of policy instruments on the policy choice are analysed.

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